

Hyperbolic geometry and Asian options

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Abstract

We first present some facts about hyperbolic geometry including the Lobacevskij formula of parallels and some basic results of hyperbolic trigonometry.

We introduce the most famous euclidean models of hyperbolic geometry, that is the Poincaré half-plane and the Poincaré disc.

In the Poincaré half-plane we give the hyperbolic coordinates and their relationship with the cartesian coordinates.

We give the hyperbolic Laplacian in the Poincaré half-plane and also, for completeness, the polar Laplacian in the Poincaré disc.

We give the stochastic representation of a Brownian motion on the Poincaré half-plane, in differential form (also in hyperbolic coordinates) and in integral form.

The generalized Bougerol formula for the stochastic representation of the hyperbolic Brownian motion is given.

The quadratic variation of the X-component of the hyperbolic Brownian motion is the integral of the geometric Brownian motion and thus is related to Asian options where the underlying asset behaves as in the Black-Scholes model.