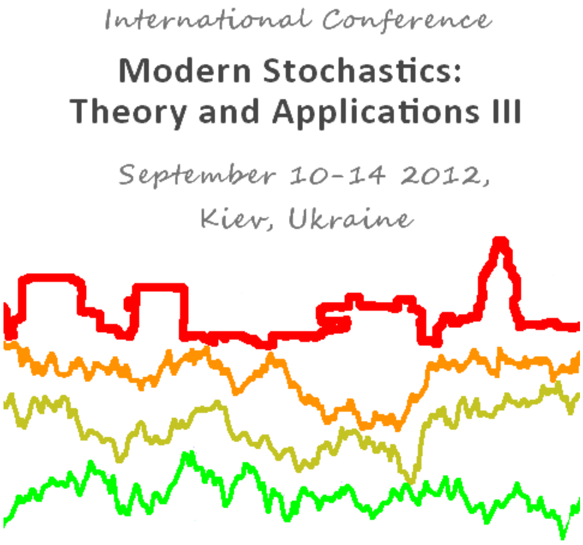
TARAS SHEVCHENKO NATIONAL UNIVERSITY OF KYIV

INTERNATIONAL CONFERENCE

MODERN STOCHASTICS: THEORY AND APPLICATIONS III

Dedicated to 100th anniversary of B.V. Gnedenko and 80th anniversary of M.I. Yadrenko

September 10-14, 2012, Kyiv, Ukraine



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CONTENTS

BORIS VLADIMIROVICH GNEDENKO[1]

(01.01.1912 - 27.12.1995)

An academician Boris Vladimirovich Gnedenko was an outstanding scientist not only in the field of pure mathematics but also in its numerous applications. Also, a substantial part of his creative heritage consists of papers on the history and pedagogy of mathematics. He revealed himself as a talented organizer of the scientific schools and the supervisor of many well-known scientists.

By virtue of Gnedenko's activity the Ukrainian school in Probability, Mathematical Statistics and their applications was founded and gained a powerful development. He made a basis for the development of computer science and computer mathematics within the Academy of Science of Ukraine. He organized in 1954 the Computing Center and supervised the work on the construction of the new generation of computing machines and algorithmic languages.

Gnedenko was born on January 1, 1912 in Simbirsk in the family of the land surveyor and the music teacher. At the age of 15 years old he completed his secondary education and entered the University of Saratov. However, he was too young to meet the official 17 years entry requirements and he was allowed to enter the university only by the special permission from A.V. Lunacharskii, the Soviet Minister of Education. He graduated from the University in 1930.

At that time one of Gnedenko's university teachers G. P. Boev took a position of the Head of the Mathematical Department at the Ivanovo-Voznesensk Textile Institute and invited Gnedenko as assistant. Working there, B.V. Gnedenko became engaged in development of mathematical methods and models for using machines in textile manufacturing. He wrote his fist works on queuing theory, which was concerned with problems of reliability of the machines used in textile manufacture. He became interested in probability theory after attending seminars organized by A.N. Kolmogorov and A.Ya. Khinchin. This period has played a significant role in his development as a scientist and a teacher.

In 1934 B.V. Gnedenko decided to resume his university studies at the postgraduate level at Moscow State University. There he investigated the limit theorems for sums of independent random variables under supervision of A.Ya. Khinchin and A.N. Kolmogorov. In 1937 B.V. Gnedenko defended his candidate thesis devoted to the theory of infinitely divisible distributions and was appointed as an assistant researcher in the Institute of Mathematics of the Moscow State University. In his candidate thesis B.V. Gnedenko established the conditions for existence of the limit distributions for sums of independent random variables and conditions of convergence to any possible limit distribution by developing the new method of accompanying infinitely divisible laws.

B.V. Gnedenko had held the positions of Associate Professor at the Department of Probability Theory at Moscow State University and Scientific Secretary of the Institute of Mathematics since autumn 1938. At that time the areas of his scientific interests were presented by asymptotic distributions for the terms of ordered sample, the structure of corresponding distributions and conditions of convergence to them. He also created the theory of corrections to the registrations by the Geiger-Müller counter.

He defended his doctoral dissertation devoted to the theory of summation of independent random variables and to the distribution of maximal term of order statistics in May 1941. During Great Patriotic War (1941-1945) his researches were concerned to solution of numerous problems related to the military defense of the Soviet Union.

In February 1945 B.V. Gnedenko was elected a corresponding member of the Ukrainian Academy of Sciences and delegated to Lviv for restitution of the teaching and research activity of Lviv University which was completely destroyed there during the war. He undertook the position of Professor and taught the courses on Mathematical Analysis, Calculus of Variations, Theory of Analytical Functions, Probability Theory, Mathematical Statistics. As for his scientific activity, he had proved in its final form the local limit theorem for independent variables indexed by a lattice terms in 1948 and started his investigations on nonparametric statistical methods. Among his disciples of that periods are O. S. Parasyuk, E. L. Yushchenko (Rvachova), V. L. Rvachov, Yu. P. Studnev, and I. D. Kvit.

In 1948 B.V. Gnedenko was elected academician of the Ukrainian Academy of Sciences. He moved to Kiev where he founded the Department of Probability Theory and Algebra at Kiev University and became its first Head. Simultaneously, he had been a Head of the Probability Department of the Institute of Mathematics of the Ukrainian Academy of Sciences. His first students in Kiev were the future academicians V. S. Korolyuk and V. S. Mikhalevich.

During that period B.V. Gnedenko created several prominent editions. Jointly with A.Ya. Khinchin, B.V. Gnedenko wrote a book "An elementary introduction to probability theory" (1946). It was republished several times in USSR with total number of copies more than half a million, and also was published in 13 foreign countries, translated into 15 languages, with the most recent edition in Russia in 2012. In 1946 one more B.V. Gnedenko's book "Essay on the history of mathematics in Russia" was published. The most remarkable is B.V. Gnedenko's classical textbook "A Course on the theory of probability"(1949), which became one of the main guidance on this subject. This textbook has gone through many editions: 3 in Ukrainian, 10 in Russian and has been translated into 11 languages. In 1949 he published a monograph, jointly with A.N. Kolmogorov, "Limit distributions for sums of independent random variables" which gained a wide international recognition and was awarded by Chebyshev Prize of the Soviet Academy of Sciences.

During Gnedenko's staying in Kiev, he attracted many young people to the study of problems in probability theory, theory of stochastic processes and mathematical statistics. He was surrounded by his disciples V.S. Korolyuk, A.V. Skorokhod, V.S. Mikhalevich, I.N. Kovalenko, M.I. Yadrenko, and many other talented mathematicians. Their main investigations were focused on limit theorems for stochastic processes and their application to the problems of mathematical statistics.

During 1953-54 B.V. Gnedenko had delivered lectures and supervised PhD students in Humboldt University in Berlin. Among his disciples of that period are such mathematicians as I. Kersten, K. Mattes, D. Koenig, G. Rossberg, W. Richter.

[1]Based on the paper by Dmitry B. Gnedenko in Electronic Journal "Reliability: Theory and Applications", Vol.2 No.4, Issue of December, 2011.

After returning to Kiev at the end of 1954 B.V. Gnedenko had organized Computer Center of Ukrainian Academy of Sciences and was in charge of the work on elaboration of the new generation of computers in USSR and creating of algorithmic languages and software for them.

In 1955 B.V. Gnedenko became a Director of the Institute of Mathematics of Ukrainian Academy of Sciences and a Chairman of the Bureau of Physics and Mathematics Department of Ukrainian Academy of Sciences. He combined this work with delivering lectures and supervising PhD students at Kiev University and at Kiev Higher Radio-engineering School (KVIRTU). B.V. Gnedenko lectured a new course on programming in the university. Based on these lectures he wrote together with V. S. Korolyuk and E. L. Yushenko the first textbook on programming "Elements of programming" in 1961. Gnedenko invited the famous algebraists V. M. Glushkov and L. A. Kaluzhnin to perform their teaching and research activity in Kiev.

Continuing his scientific research, B.V. Gnedenko developed a number of applied approaches in operation research (queuing theory and mathematical theory of reliability), application of the mathematical methods in medicine and methods for calculation of electrical networks for industry. Gnedenko formed the scientific school in queuing theory from his disciples: I. N. Kovalenko, T. P. Marjanovich, I. V. Yarovitskiy, S. M. Brodi and others. In 1959 he published the book "Lectures on queuing theory" which was based on his lectures in KVIRTU. Further on he wrote jointly with I. N. Kovalenko a monograph "Introduction into the queuing theory" (1966) which has 5 editions. During that period he had designed together with E. A. Shkabara and N. M. Amosov the first electrical heart disease diagnostic device implemented in 1960.

In 1960 Gnedenko left Kiev for Moscow and resumed his work at the Mechanics and Mathematics Faculty of Moscow University.

There he organized scientific school in mathematical methods in reliability theory inviting to this activity such prominent mathematicians as Yu.K. Belyaev, A.D. Solovjev, Ya.B. Shor, I.N. Kovalenko, V.A. Kashtanov, I.A. Ushakov among others. In 1965 the joint monograph "Mathematical methods in reliability theory" by B.V.Gnedenko, Yu.K.Belyaev and A.D. Solovjev was published. This monograph got a wide recognition in the USSR and abroad. For the research in this area B.V.Gnedenko received the USSR State Award in 1979 (within the group of researches).

Gnedenko continued to study the limit theorems for sums of independent random variables with random number of terms. For these results he was awarded by the Lomonosov Prize in 1985 and by the Ministry of Higher Education Award in 1986.

He paid much attention to the problems of mathematical education at secondary and higher school, and also to the history of mathematics and probability theory, in particular. Gnedenko published a number of books and many papers on these topics.

At the beginning of 1966 B.V. Gnedenko took a position of the Head of the Probability Theory Department after A.N. Kolmogorov. He carried out these duties during 30 years till the end of his life. At the beginning of 90th Gnedenko initiated and implemented the economic specialization at the Mechanics and Mathematics Faculty of Moscow University. This allowed to qualify specialists in actuarial and financial mathematics.

Diversity of Gnedenko's research interests and activity scope one can be seen from the list of his publications which consists of about 1000 items. Being the prominent scientist and pedagogue, a person of high intellectual capacities, B.V. Gnedenko was very active in popularization of scientific knowledge, he devoted a lot of his time and efforts to the work at the civil society organization "Knowledge" ("Znanie") and held leading positions there.

Gnedenko was a member of the Editorial Boards of many soviet and foreign scientific journals, a member of the Royal Statistical Society (UK), the Honorary Doctor of Berlin and Athens universities.

Gnedenko's ideas and scientific heritage has had a great impact in various fields of mathematical science, his disciples and followers continue and develop the researches started by him. Among them there are many academicians and corresponding members of academies, professors of different universities.

MYKHAILO IOSYPOVYCH YADRENKO[1]

(16.04.1932 - 28.09.2004)

Professor Mykhailo Iosypovych Yadrenko was a prominent mathematician and pedagogue, Corresponding Member of the Ukrainian National Academy of Sciences, Honored Personality of Science and Technology of Ukraine.

It is impossible to imagine the theory of random fields, one of the main branches of probability theory, without M.I. Yadrenko's works, which founded numerous new directions of this theory. It is also hard to overestimate his contribution to the development of mathematical education in the Ukraine and training of highly qualified specialists in probability and statistics.

M.I. Yadrenko was born on April 16, 1932 in the village of Drimailivka, Nizhyn district, Chernihiv region. In 1950-1955, he was a student at the Department of Mechanics and Mathematics at the Kiev University, where he attended lectures of prominent mathematicians and teachers such as N.N. Bogolyubov, B.V. Gnedenko, and I.I. Gikhman. Under their guidance, M.I. Yadrenko began his scientific studies and published his first scientific work devoted to the investigation of properties of random walks.

In 1955-1958, he was a postgraduate student at the Kiev University. In those years, M.I. Yadrenko began his studies of homogeneous and isotropic random fields. These investigations were presented in his Candidate-Degree (Ph.D.) Thesis entitled "Some problems in the theory of random fields", supervised by I. I. Gikhman.

After his postgraduate studies, M.I. Yadrenko started to work at the Department of Mathematical Analysis and Probability Theory at the Kiev University. At the same time, he devoted much energy to the development of mathematical education at secondary schools, organization of mathematical competitions, and publication of contemporary textbooks on elementary mathematics and combinatorial analysis and books of problems of mathematical competitions.

In 1966, M.I. Yadrenko became the Head of the Department of Probability Theory and Mathematical Statistics and headed it for more than 33 years. Under his guidance, the researchers of the department carried out investigations in the spectral theory of random fields (in particular, the Doctoral-Degree Thesis of M.I. Yadrenko was devoted to this theory), asymptotic methods in probability theory, the theory of stochastic differential equations, and applied problems in probability theory and mathematical statistics. The Department of Probability Theory and Mathematical Statistics was repeatedly distinguished as the best department of the Kiev University.

In 1970, the journal *Teoriya Veroyatnostei i Matematicheskaya Statistika (Probability Theory and Mathematical Statistics)* was founded on M.I. Yadrenko's initiative (since 1992, this journal is published in Ukrainian). This journal has played an important role in the development of the world-known Kiev school in probability theory. Since 1974, the English version of this journal has been published by the American Mathematical Society. In 1969, M.I. Yadrenko founded another periodical edition, a collection of popular-science works *U Sviti Matematyky (In the World of Mathematics).* In 1995, *U Sviti Matematyky* was transformed into a popular scientific, methodical, and historical journal. This journal is intended for a broad circle of readers interested in mathematics from scholars to professional mathematicians.

As a scientist, M.I. Yadrenko is well known in the world for his works devoted to the theory of random fields and their statistical analysis. Together with A.M. Yaglom, M.I. Yadrenko is considered to be a founder of the spectral theory of random fields. In 1961, M.I. Yadrenko and A.M. Yaglom derived the spectral representation of a mean-square continuous homogeneous and isotropic random field. This result is contained in all well-known handbooks and monographs dealing with the theory of random fields. M.I. Yadrenko created effective methods of solving statistical problems for random fields: extrapolation, filtration, interpolation, and estimation of the regression coefficients. M.I. Yadrenko was one of the first to study analytic properties of random fields. He proved new theorems containing general conditions of the sample continuity with probability one for random fields and calculated the modulus of continuity for these fields. Along with random fields defined on finite-dimensional spaces, M.I. Yadrenko also studied those defined on Hilbert spaces. He introduced the concept of an isotropic field on the sphere in a Hilbert space and the notion of a homogeneous and isotropic random field on the whole Hilbert space and obtained their spectral representations. These results enabled M.I. Yadrenko to prove an ergodic theorem for random fields. Spectral representations due to M.I. Yadrenko have been used in modeling random fields and in studying some problems related to the invariance principle. M.I. Yadrenko was the first to study conditions providing sample continuity of random fields on compact sets in a Hilbert space. He founded the theory of Markov random fields-a new direction in the theory of random fields. Later, the theory of Markov random fields was further developed in works related to problems of statistical physics and quantum field theory.

M.I. Yadrenko is the author of more than 200 scientific papers. A survey of the results obtained by Yadrenko in the theory of random fields can be found in the paper of Buldygin, Kozachenko, and Leonenko "On Yadrenko's works in the theory of random fields", Ukr. Mat. Zh., 44, No. 11 (1992). One should especially note the Yadrenko's monograph *"Spectral Theory of Random Fields"*, Vyshcha Shkola, Kiev (1980), which became a handbook for experts in random fields. This book was awarded the Prize of Ukrainian Ministry of Education, and its English version was published in 1983 in the USA.

M.I. Yadrenko obtained numerous important results in various branches of applied probability theory such as optimal methods for quality control in mass production, statistical modeling of noises in semiconductors, statistical analysis of generators of random numbers, statistical problems of reliability theory, and statistical models of distributions with random intensity).

For a series of works in the theory of random fields, M.I. Yadrenko was awarded the Krylov Prize of the Ukrainian National Academy of Sciences.

M.I. Yadrenko was the founder and the leader of the well-known scientific school in random fields and processes. Among his disciples, there are 45 Candidates of Science. 11 of his disciples became Doctors of Sciences; these are V.V. Anisimov, Corresponding Member of the Ukrainian National Academy of Sciences, and professors V.L. Girko, Yu.V. Kozachenko, M.M. Leonenko, Yu.D. Popov, D.S. Silvestrov, N.M. Zinchenko, M.P. Moklyachuk, O.I. Klesov, A.B. Kachyns'kyi, I.V. Matsak.

[1]Based on the paper "Mykhailo Iosypovych Yadrenko", Theory of Probability and Mathematical Statistics, No. 74, 2007.

M.I. Yadrenko was also a remarkable teacher. His lectures were distinguished by mathematical rigorousness, high scientific level, and clarity of presentation. Many known experts in probability theory and mathematical statistics learned these branches of mathematics at his lectures. M.I. Yadrenko is the author of 24 textbooks on different branches of mathematics, among which one should mention *Probability Theory and Mathematical Statistics* [Vyshcha Shkola, Kiev (1988)] (written together with I. I. Gikhman and A. V. Skorokhod) and a remarkable book of problems in probability theory (written together with A. V. Skorokhod, A. Ya. Dorogovtsev, and D. S. Sil'vestrov), the English version of which was published in 1997 in the USA.

In 1995, for the first time in Ukraine, M.I. Yadrenko began to deliver lectures on actuarial mathematics and the theory of insurance risk. Together with his disciples M.M. Leonenko, Yu. S. Mishura and V. M. Parkhomenko, Yadrenko published the first Ukrainian textbook on econometrics and contemporary financial and actuarial mathematics. M.I. Yadrenko is one of the initiators of the introduction of the new mathematical speciality "Statistics" in Ukraine. In 1997-2001, he headed the Statistical Aspects of Economics global international project within the framework of the TEMPUS-TACIS program of the European Community.

One should especially note the tremendous contribution of M.I. Yadrenko to the training of talented young mathematicians. For more than 40 years, he was an organizer of mathematical circles for scholars and school mathematical competitions at different levels, headed the Jury of All-Ukrainian mathematical competitions for scholars and students, he was the editor of *U Sviti Matematyky*, an unparalleled journal for scholars. For many years, he also delivered TV lectures on mathematics for scholars.

All the facts presented above do not exhaust the versatile activities of M.I. Yadrenko. He was the Vice-President of the Ukrainian Mathematical Society, a member of the bureau of the Department of Mathematics at the Ukrainian National Academy of Sciences, the Editor in Chief of the *Prykladna Statystyka, Aktuarna ta Finansova Matematyka* scientific journal, a deputy editor-in-chief of the *Teoriya Imovirnostei ta Matematychna Statystyka* journal, and a member of the Editorial Board of the *Random Operators and Stochastic Equations* international journal.

For his scientific and pedagogical activities, M.I. Yadrenko was awarded State Prize of Ukraine (2003).

All disciples, colleagues and friends of M.I. Yadrenko always treated him with love and deep respect, taking him as an example of a person, scientist, and teacher.

FRACTAL ANALYSIS

ON FINE FRACTAL PROPERTIES OF MEASURES WITH INDEPENDENT Q^*-DIGITS AND DP-TRANSFORMATIONS

M. Ibragim, G. Torbin

Let $Q^* = ||q_{ik}||$ be a given stochastic matrix such that $q_{ik} > 0$, $\sum_{i=0}^{s-1} q_{ik} = 1$, $\prod_{k=1}^{\infty} \max_i q_{ik} = 0$, and let $\Delta_{\alpha_1(x)\alpha_2(x)\ldots\alpha_k(x)\ldots}$ be the Q^*-expansion of a real number x from the unit interval ([1]). It is known that condition $\inf_k q_{0k} > 0$, $\inf_k q_{(s-1)k} > 0$ implies the faithfulness for the family $\Phi(Q^*)$ of Q^*-cylinders. Firstly we show that this condition is not necessary for the faithfulness of $\Phi(Q^*)$.

Let $\xi = \Delta_{\xi_1\xi_2\ldots\xi_k\ldots}$ be the random variable with independent Q^*-digits ξ_k, taking values $0, 1, \ldots, s-1$ with probabilities $p_{0k}, p_{1k}, \ldots, p_{(s-1)k}$. Fine fractal properties of μ_ξ were studied in [2]. Let us recall that an automorphism F of $[0, 1]$ is said to be DP if $\dim_H(E) = \dim_H(F(E))$, $\forall E \subset [0, 1]$. If F is continuous, then F is a distribution function of some r.v. ψ or it is of the form $1 - F_\psi$. In the talk we present new results on DP-properties of F_ξ.

Theorem 1. *Let* $\inf_{i,k}\{q_{ik}, p_{ik}\} > 0$. *Then* F_ξ *preserves the Hausdorff-Besicovitch dimension on the unit interval if and only if* $\dim_H \mu_\xi := \inf\{\dim_H E, \mu(E) = 1\}$ *is equal to* 1.

A number of examples and counterexamples will be presented. In particular we show that even under condition $\inf_{i,k} q_{ik} > 0$ there exist measures μ_ξ of full Hausdorff-Besicovitch dimension such that F_ξ are not DP.

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National Pedagogical University, Institute for Mathematics, Kyiv, Ukraine
E-mail address: muslemhussen1978@yahoo.com, torbin7@gmail.com, torbin@iam.uni-bonn.de

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ON NECESSARY AND SUFFICIENT CONDITION FOR INFINITE BERNOULLI CONVOLUTIONS TO BE $\widetilde{Q}$-MEASURES

G. Ivanenko, G. Torbin

Let μ_ξ be the distribution of $\xi = \sum_{k=1}^{\infty} \xi_k a_k$, where $a_k \geq 0$, $\sum_{k=1}^{\infty} a_k < +\infty$ and ξ_k are independent Bernoulli random variables. Measures of such a type were studied intensively during last 80 years. Properties of μ_ξ are well known now for the case where $a_k \geq r_k := \sum_{j=k+1}^{\infty} a_j$ for all k large enough. Main problems appear for the case of large overlaps, i.e. if λ-almost all real numbers from the spectrum S_ξ have continuum many different expansions of the form $\sum_{k=1}^{\infty} \varepsilon_k a_k$, $(\varepsilon_k \in \{0, 1\})$. Special partial cases of such Bernoulli convolutions were studied by S. Albeverio, Ya. Goncharenko, G. Ivanenko, M. Lebid, M. Pratsiovytyi, G. Torbin. All measures which were studied by the above authors belong to the measures with independent $\widetilde{Q}$-digits [1]. We describe completely all infinite Bernoulli convolutions which are measures with independent $\widetilde{Q}$-digits.

Let $I_n := \{x : x = \sum_{k=1}^{n} \varepsilon_k a_k, \varepsilon_k \in \{0, 1\}\}$, $\lambda_n := \min\{|x - y|, x, y \in I_n, x \neq y\}$.

Theorem 1. *An infinite Bernoulli convolution* μ_ξ *belongs to the family of measures with independent* $\widetilde{Q}^*$*-digits if and only if condition* $\lambda_n \geq r_n$ *holds for an infinite number of indices* n.

We also present necessary and sufficient conditions for such measures to be absolutely resp. singularly continuous, and some results on fractal properties of μ_ξ and its spectrum.

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National Pedagogical University, Institute for Mathematics, Kyiv, Ukraine
E-mail address: anna ivanenko@ukr.net, torbin7@gmail.com, torbin@iam.uni-bonn.de

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ON FRACTAL PROPERTIES OF THE SET OF QUASINORMAL CONTINUED FRACTIONS

Yu. Kulyba, G. Torbin

Let $x = [0; \alpha_1(x)\alpha_2(x)\ldots\alpha_k(x)\ldots]$ be the classical continued fraction expansion of $x \in (0,1)$, and let $\xi = [0; \xi_1\xi_2\ldots\xi_k\ldots]$ be the random variable with independent identically distributed c.f.-digits taking values $1, 2, \ldots$ with probabilities $p_1, p_2, \ldots$. Let $N_i(x,k)$ be the number of the digit «i» among the first k c.f.-digits of $x \in (0,1)$, and let $\nu_i(x) := \lim\limits_{k\to\infty} \frac{N_i(x,k)}{k}$ be the asymptotic frequency of «i» in the c.f.-expansion of x. It is well known that for λ-almost all real numbers $x \in (0,1)$ one has

$$\nu_i(x) = \frac{1}{\ln 2} \cdot \ln \frac{(i+1)^2}{i(i+2)}, \quad \forall i \in N. \tag{1}$$

Those x for which (1) holds are said to be c.f.-normal. A real number x is said to be c.f.-quasinormal if $\nu_i(x)$ exists for any $i \in N$ and there exists $m \in N$ such that

$$\nu_m(x) \neq \frac{1}{\ln 2} \cdot \ln \frac{(m+1)^2}{m(m+2)}.$$

In [1] it has been proven that there exists an absolute constant ε_0 such that $\dim_H \mu_\xi \leq 1 - \varepsilon_0$ for any above defined r.v. ξ. So, the standard probabilistic approach fails to prove the superfractality for the set of c.f.-quasinormal numbers. Nevertheless we prove the following theorem.

Theorem 1. *The Hausdorff-Besicovitch dimension of the set of quasinormal continued fractions is equal to* 1.

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National Dragomanov Pedagogical University, Institute for Mathematics, Kiev, Ukraine
E-mail address: kuliba9@gmail.com, torbin7@gmail.com, torbin@iam.uni-bonn.de

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ON LEBESGUE STRUCTURE AND FRACTAL PROPERTIES OF SOME GENERALIZED INFINITE BERNOULLI CONVOLUTIONS

M. Lebid

Let $\mu_\xi = \mu$ be the probability measure corresponding to the random variable

$$\xi = \sum_{k=1}^{\infty} \xi_k a_k,$$

where $\sum_{k=1}^{\infty} a_k$ is a positive convergent series, and ξ_k are independent random variables taking values 0 and 1 with probabilities p_{0k} and $p_{1k} = 1 - p_{0k}$ correspondingly.

We consider the case where the sequence $\{a_k\}$ satisfies following two conditions:
$(*)$ $a_k < r_k$ *holds for the infinite number of* k;
$(**)$ *for any* $k \in \mathbb{N}$ *there exists a number* $s_k \in \{0,\ 1,\ 2,\ ...\}$ *with* $a_k = a_{k+1} = ... = a_{k+s_k} \geqslant r_{k+s_k}$. In such a case we deal with "large overlaps" ([2]). Let $\{k_n\}$ be the sequences of positive numbers with the following property: $i \in \{k_n\}$ if and only if $s_i = 0$. Let, $l_n = k_n - k_{n-1}$, $k_0 = 0$.

Theorem 1. *Suppose* ξ *satisfies* $(*)$ *and* $(**)$. *Then the random variable* ξ *is singularly distributed and the Hausdorff dimension of its spectrum (minimal topological support of r.v.) is equal to*

$$\dim_H(S_\xi) = \varliminf_{n\to\infty} \left(\frac{\sum_{j=1}^{n} \ln(l_j + 1)}{-\ln r_{k_n}} \right).$$

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National Dragomanov Pedagogical University, Kyiv; Bielefeld University
E-mail address: mykola.lebid@gmail.com

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DISTRIBUTION OF RANDOM VARIABLE REPRESENTED BY BINARY FRACTION WITH TWO REDUNDANT DIGITS 2 AND 3 HAVING THE SAME DISTRIBUTION

O.P. Makarchuk

Let ξ_k be a sequence of independent random variables taking the values 0,1,2,3 with probabilities p_0, p_1, p_2, p_3 respectively. The random variable

$$\xi = \sum_{k=1}^{\infty} \xi_k 2^{-k}$$

is called a random variable represented by binary fraction with two redundant digits 2 and 3 having the same distribution.

Theorem 1. **([1])** *Random variable ξ has a pure distribution, moreover*

1) it has a discrete distribution if and only if $p_{\max} = \max\limits_{0\leqslant i\leqslant 3} p_i = 1$,

2) if $\begin{cases} p_0 - p_1 + p_2 - p_3 \neq 0 \\ (p_0-p_2)^2 + (p_1-p_3)^2 \neq 0 \\ p_{\max} = \max\limits_{0\leqslant i\leqslant 3} p_i \neq 1 \end{cases}$ *, then it has a singular distribution,*

3) if $(p_0-p_2)^2 + (p_1-p_3)^2 = 0$, then it has an absolutely continuous distribution.

We solve the problem on the type of distribution of the random variable ξ for the case $p_0 - p_1 + p_2 - p_3 = 0$.This problem is unsolved for a long time, so all approaches to its solution have a scientific interest.

Theorem 2. *If $p_0 - p_1 + p_2 - p_3 = 0$, then the random variable ξ is of absolutely continuous distribution.*

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National Pedagogical University, Pyrogova Str. 9, 01030 Kiev, Ukraine

E-mail address: makolpet@gmail.com

ON FAITHFUL AND NON-FAITHFUL COVERING FAMILIES OF CYLINDERS GENERATED BY THE Q_∞-EXPANSION

R. Nikiforov[1], G. Torbin[2]

Let $Q_\infty = (q_0, q_1, \ldots, q_k, \ldots)$ be a stochastic vector with $q_i > 0$. For any $x \in [0,1)$ there exists a unique sequence $\{\alpha_k(x)\}$ of non-negative integers such that

$$x = \beta_1(x) + \sum_{k=2}^{\infty} \beta_k(x) \cdot \prod_{j=1}^{k-1} q_{\alpha_j(x)} =: \Delta_{\alpha_1(x)\alpha_2(x)\ldots\alpha_k(x)\ldots}, \tag{1}$$

where $\beta_k(x) = \sum\limits_{i=0}^{k-1} q_i$ with $\sum\limits_{i=0}^{-1} q_i := 0$. Expression (1) is said to be the polybasic Q_∞-expansion for real numbers ([1]). Let Φ be a covering system consisting of Q_∞-cylinders of $[0,1)$ and let $\dim_H(E,\Phi)$ be the Hausdorff dimension of a set $E \subset [0,1)$ w.r.t. Φ. A system Φ is said to be faithful if $\dim_H(E,\Phi) = \dim_H(E), \forall E \subset [0,1]$.

Theorem 1. *If there exist real numbers q_*, q^* such that $0 < q_* \leq \frac{q_i}{q_{i-1}} \leq q^* < 1, \quad \forall i \in \mathbb{N}$, then Φ is faithful.*

Theorem 2. *If there exists a positive integer $m_0 > 1$ and real numbers A and B such that $\frac{A}{(i+1)^{m_0}} \leq q_i \leq \frac{B}{(i+1)^{m_0}}, \forall i \in \mathbb{N}$, then Φ is non-faithful.*

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[1] National Pedagogical University, Kiev, Ukraine
E-mail address: rnikiforov@gmail.com

[2] National Pedagogical University, Institute for Mathematics, Kyiv, Ukraine
E-mail address: torbin7@gmail.com, torbin@iam.uni-bonn.de

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THE RANDOM ALTERNATING LÜROTH SERIES WITH ELEMENTS WHICH FORMS A HOMOGENEOUS MARKOV CHAIN

M.V. Pratsiovytyi, Yu.V. Khvorostina

In the report we offer results of studying of the properties of a random variables $\xi = \Delta^{\widetilde{L}}_{\eta_1\eta_2\ldots\eta_n\ldots}$ represented by the alternating Lüroth series (see [1]) with elements η_k are random variables which forms a homogeneous Markov chain and taking the values positive integers with the initial probabilities $p_1, p_2, \ldots, p_i, \ldots$ and the transition probability matrix $\|p_{ij}\|,\ i,j \in N,\ p_i > 0,\ p_{ij} \geq 0$. We have derived the analytical expression of the probability distribution function $F_\xi(x)$ and of its derivative at a point. We found the intervals of monotonicity of the distribution function depending on elements of the transition probability matrix $\|p_{ij}\|$. We investigated some metric, topological and fractal properties of the spectrum of distribution of the random variable ξ (the spectrum is a minimal closed support of distribution).

Theorem 1. *The spectrum S_ξ of distribution of the random variable ξ is the union of set* $A = \{x :\ x \in (0;1),\ p_{a_k(x)a_{k+1}(x)} > 0 \quad \forall k \in N\}$ *and rational points of* $(0;1]$ *which is the limit for A. Lebesgue measure of the spectrum is zero if the transition probability matrix $\|p_{ij}\|$ has zero.*

Theorem 2. *The probability distribution of the random variable ξ has atoms if and only if there exists either tuple* $(a_1, a_2, ..., a_k, ...)$ *such that* $p_{a_1(x)} \prod\limits_{k=1}^{\infty} p_{a_k(x)a_{k+1}(x)} > 0$.

The report contains the results of research of Lebesgue structure of the random variable ξ, the conditions of preservation of Hausdorff-Besicovitch dimension of the probability distribution function, the comparative analysis of the solution of the basic probability problems of the elementary continued fractions and alternating Luroth series representations for real numbers.

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Drahomanov National Pedagogical University, Physics and Mathematics Institute, Pyrohova str. 9, Kyiv, Ukraine
E-mail address: prats4@yandex.ru, khvorostina13@mail.ru

PROBABILITY DISTRIBUTIONS ON GRAPHS OF ONE CLASS OF NON-DIFFERENTIABLE FUNCTIONS

M.V. Pratsiovytyi, N.A. Vasilenko

Let s be a fixed odd positive integer, $s > 3$, $A = \{0, 1, \ldots, s-1\}$, $m = \frac{s-3}{2}$,

$$\gamma(\alpha) = \begin{cases} 0, & \text{if} \quad \alpha = 0, \\ 1, & \text{if} \quad \alpha \in A \setminus \{0, s-1\}, \\ 2, & \text{if} \quad \alpha = s-1. \end{cases}$$

For any sequence $(\alpha_n) \in L \equiv A^\infty = A \times A \times \ldots$ define a sequence (c_k)

$$c_1 = 0, c_k = \begin{cases} c_{k-1}, & \text{if } \alpha_{k-1} \in A \setminus \{2, 4, \ldots, s-3\}, \\ 1 - c_{k-1}, & \text{if } \alpha_{k-1} \notin A \setminus \{2, 4, \ldots, s-3\}. \end{cases}$$

Define a function, with the argument given in the form

$x = \varphi_{\alpha_1} + \sum_{i=2}^{\infty} \left[\varphi_{\alpha_i} \cdot \prod_{j=1}^{i-1} q_{\alpha_j}\right] \equiv \Delta^{Q_s}_{\alpha_1\,\alpha_2\,\cdots\alpha_k\cdots},\ \ \alpha_k \in A,$

where $Q_s = \{q_0, q_1, \ldots, q_{s-1}\}$, $q_i > 0$, $\sum_{i=0}^{s-1} q_i = 1$, $\varphi_0 = 0$, $\varphi_k = \sum_{i=1}^{k-1} q_i$,
and value of function has the following Q'_3-representation

$f(x)=\Delta^{Q_3}_{\beta_1\,\beta_2\,\ldots\beta_k\ldots}\equiv\psi_{\beta_1}+\sum_{i=2}^{\infty}\left[\psi_{\beta_i}\cdot\prod_{j=1}^{i-1}q'_{\beta_j}\right],\quad \beta_k\in B\equiv\{0,1,2\},$
where $Q'_3=\{q'_0,q'_1,q'_2\}$, $q'_i>0$, $q'_0+q'_1+q'_2=1$, $\psi_0=0$, $\psi_k=\sum_{i=1}^{k-1}q'_i$,

$$\beta_k=\begin{cases}\gamma(\alpha_k), & \text{if } c_k=0,\\ 2-\gamma(\alpha_k), & \text{if } c_k\neq 0.\end{cases}$$

In the report are given results of structural, metric-topological and fractal properties of the function f random argument $\xi=\Delta^{Q_s}_{\eta_1\eta_2\ldots\eta_k\ldots}$, where η_k - random variables with the following distribution $P\{\eta_k=i\}=p_{ik}\geq 0$, $p_{0k}+\ldots+p_{(s-1)k}=1$.

Drahomanov National Pedagogical University, Kyiv, Ukraine
E-mail address: prats4@yandex.ru, nata_va@inbox.ru

FRACTAL AND PROBABILISTIC THEORIES OF REPRESENTATION OF NUMBERS BY SYLVESTER SERIES

M.V. Pratsiovytyi, M.V. Zadniprianyi

Theorem 1. *For any real number $x\in(0,1]$ there exists a unique sequence of positive integers (q_k) such that $q_1\geq 2$, $q_{n+1}\geq q_n(q_n-1)+1$, $n\in N$ and*

$$x=\frac{1}{q_1}+\frac{1}{q_2}+\ldots+\frac{1}{q_n}+\ldots=\Delta_{q_1q_2\ldots q_n\ldots}. \tag{1}$$

One can rewrite Equation (1) in a more convenient form: $x=\Delta^S_{g_1g_2\ldots g_n\ldots}$, where $g_1=q_1-1$, $g_{k+1}=q_{k+1}-q_k(q_k-1)$, $k\in N$. We call this form by *S-representation* of number x and $g_k=g_k(x)$ by its *k-th S-symbol.* Specific feature of the representation of the positive real number by Sylvester series (1) is the following: the real number is modeled from the positive integers, and its formal S-representation uses an infinite alphabet, which is a set of positive integers.

Lemma 2. *If a random variable has an uniform distribution on $[0,1]$, then its S-symbols are dependent random variables and their dependence is more complicated than Markovian one.*

Theorem 3. *The probability distribution function of the random variable $\xi=\Delta^S_{\tau_1\tau_2\ldots\tau_n\ldots}$ with independent S-symbols τ_n such that $P\{\tau_k=i\}=p_{ik}\geq 0$, $i\in N$, $k\in N$, has a Q^*_∞-representation. If $p_{ik}>0$ for all i and k, then the probability distribution function is strictly increasing. If there are zeros in the matrix $\|p_{ik}\|$, then the set of points of growth has fractal properties.*

In the talk we study the Lebesgue structure of ξ and the structure of singular distribution in the case of the singularity of ξ. In particular, we prove the purity and provide criterions for the probability distribution to be of each pure type. The properties of the random variable which S-symbols form a homogeneous Markov chain as well as the random variables (τ_{2k-1}) are independent and (τ_{2k}) form a Markov chain are also considered.

Dragomanov National Pedagogical University
E-mail address: zadnipryanyi.maksim@gmail.com

ON ONE CLASS OF DISTRIBUTIONS OF RANDOM VARIABLES

Yu. Ralko

Let (d_n) be a fixed periodic sequence of positive integers greater than 1. Let $(d_1d_2\ldots d_m)$ be a period of this sequence.

We consider the random variable

$$\xi=\sum_{k=1}^{\infty}\frac{\tau_k}{d_1d_2\ldots d_k},$$

where τ_k is a sequence of independent random variables with probability distributions $P\{\eta_{km+j}=i\}=p_{ij}$ for any $i=\overline{0,d_j-1}$, $j=\overline{1,m}$, $k\in N$.

Theorem 1. *If there exists $p_{ij}\neq\frac{1}{d_j}$, then ξ has a singular distribution of Cantor type if the matrix $\|p_{ik}\|$ have zeros, and Salem type if the matrix have not zeroes.*

In the talk we discuss fractal properties of the spectrum of distribution of the random variable ξ (i.e., set of points of growth of the probability distribution function), in particular, self-affine properties of graph of the probability distribution function of ξ and its integral properties. We describe the topological and metric structure of the spectrum of ξ if it is of singular distribution, and deformation of the fractal dimension of subsets of spectrum if the probability distribution function of ξ is strictly increasing.

Theorem 2. *If the sequence (d_n) is bounded, then for determination of the Hausdorff–Besicovitch dimension of the spectrum of the random variable ξ it is enough to use coverings by cylindrical intervals corresponding to representation of numbers with the sequence (d_n).*

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Dragomanov National Pedagogical University, Kyiv, Ukraine
E-mail address: yulya-ralko@yandex.ru

TOPOLOGICAL, METRIC AND FRACTAL PROPERTIES OF THE SET WITH PARAMETER, THAT THE SET DEFINED BY S-ADIC REPRESENTATION OF NUMBERS

S.O. Serbenyuk

Let we have fixed integer number $s > 2$, fixed set

$$N^1_{s-1} \equiv \{1, 2, ..., s-1\} \subset A \equiv \{0, 1, 2, ..., s-1\}$$

and sequence space

$$L \equiv (N^1_{s-1})^\infty \equiv (N^1_{s-1}) \times (N^1_{s-1}) \times (N^1_{s-1}) \times ....$$

The set $\mathbb{S}_{(s,u)}$ define in the following way

$$\mathbb{S}_{(s,u)} = \left\{ x : x = \sum_{k=1}^{\infty} \frac{\alpha_k - u}{s^{\alpha_1 + ... + \alpha_k}} + \frac{u}{s-1}, \forall (\alpha_n) \in (N^1_{s-1})^\infty, \alpha_n \neq u, 2 < s \in \mathbb{N} \right\},$$

which equivalent

$$\mathbb{S}_{(s,u)} = \left\{ x : x \equiv \Delta^s_{\underbrace{u...u}_{\alpha_1 - 1}\alpha_1 \underbrace{u...u}_{\alpha_2 - 1}\alpha_2 ... \underbrace{u...u}_{\alpha_n - 1}\alpha_n ...}, \forall (\alpha_n) \in (N^1_{s-1})^\infty, \alpha_n \neq u, u \in A, s > 2 \right\},$$

where $\Delta^s_{\alpha_1\alpha_2...\alpha_n...}$ — s-adic representation of numbers from $[0, 1]$.

Theorem 1. *The set $\mathbb{S}_{(s,u)}$ has the following properties:*

(1)
$$\bigcap_{u=0}^{s-1} \mathbb{S}_{(s,u)} = \{\Delta^s_{(1)}\}.$$

(2) *continuous, nondense, Lebesgue zero-measure, perfect and self-similar fractal with Hausdorff-Besicovitch dimension $\alpha_0(\mathbb{S}_{(s,u)})$, which calculates by formula:*
$$\sum_{p_i \neq u, p_i \in N^1_{s-1}} \left(\frac{1}{s}\right)^{p_i \alpha_0} = 1.$$

Institute of Mathematics NAS of Ukraine
E-mail address: simon6@ukr.net

PACKING DIMENSION AND PACKING DIMENSION PRESERVING TRANSFORMATIONS

Alexander V. Slutskiy

We study packing dimension ($dim_P(\bullet)$) and packing dimension preservations (PDP). One can find a definition and basic properties of $dim_P(\bullet)$ in [1] and [2]. Note that $dim_P(\bullet)$ is countably stable, i.e. $dim_P \bigcup E_i = \sup_i dim_P E_i$.

Let (X, ρ) be a metric space. An automorphism f of X is called a PDP transformation if

$$dim_P(f(E)) = dim_P(E), \forall E \subset X$$

Basic properties of PDP-transformations in metric spaces:

(1) PDP-transformations form a group with respect to operation "composition";
(2) Isometries are PDP-transformations;
(3) Similarity transformations arePDP-transformations;
(4) Bi-Lipshitz transformations are PDP-transformations. So, any affine transformation is in PDP-group.

Theorem 1. *Any projective transformation in $\mathbb{R}^1$ preserves the Hausdorff-Besicovitch dimension [3] and packing dimension.*

Theorem 2. *If E is a compact self-similar set in $\mathbb{R}^n$ that satisfying an open set condition then its packing dimension is equal to its self-similar dimension.*

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National Dragomanov Pedagogical University, Kyiv, Ukraine
E-mail address: angell2006@ukr.net

SINGULAR PROBABILITY MEASURES AND THEIR FRACTAL ANALYSIS: MOTIVATIONS, ACHIEVEMENTS, APPLICATIONS AND OPEN PROBLEMS

G. Torbin

During the talk we shall discuss several groups of problems which are important for the theory of singularly continuous probability measures (SCPM) and the theory of fractals.

Firstly, we shall analyze a role of singular measures from different points of view and present some probabilistic schemes naturally leading to singular probability distributions.

Secondly, we shall discuss

(1) Methods of proving the singularity resp. absolute continuity of probability measures;
(2) Classifications of singular measures in $\mathbb{R}^1$ and $\mathbb{R}^n$;
(3) Different approaches to the study of fractal properties of SCPM;
(4) Interplay between theory of SCPM, theory of fractals and dynamical systems.

Thirdly, we shall discuss already existing and possible applications of SCPM and concentrate on applications related to the metric theory of different symbolic expansions, fractal geometry and dynamical systems.

Finally, we shall discuss some open problems related both to the theory of SCPM and the theory of fractals. A special attention will be paid to infinite Bernoulli convolutions, SCPM on subsets of non-normal numbers w.r.t. different expansions, faithfulness of Vitaly coverings generated by different expansions of reals and their relations to fine fractal properties of underlying SCPM, constructions of operators with singular continuous spectra and theory of DP- and PDP-transformations.

National Dragomanov Pedagogical University, Institute for Mathematics, Kyiv, Ukraine
E-mail address: torbin7@gmail.com, torbin@iam.uni-bonn.de

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PROPERTIES OF DISTRIBUTIONS OF RANDOM VARIABLES RELATED TO DIFFERENT SYSTEMS OF REPRESENTATION

I.V. Zamriy, Yu.Yu. Sukholit

For any number $x \in [0;2]$ there exists a sequence (α_k), $\alpha_k \in \{0,1,2,3,4\}$, such that $x = \frac{\alpha_1}{3} + \frac{\alpha_2}{3^2} + \ldots + \frac{\alpha_k}{3^k} + \ldots = \Delta_{\alpha_1\alpha_2\ldots\alpha_k\ldots}$. This expansion is called a 3-5-adic expansion (representation) of number x. Almost all numbers $x \in [0;2]$ have a continuum set of different 3-5-adic representations. Let us choose one economical and convenient representation for developing the metric and probabilistic number theory.

Definition 1. 3-5-adic representation $\Delta_{\alpha_1\alpha_2\ldots\alpha_k\ldots}$ of number $x \neq 2$ is called *canonical*, if the following conditions are fulfilled simultaneously:
1. it does not contain a period (2);
2. there exists a number k such that
2.1. $\alpha_{k+j} \in \{0,1,2\}$ for any $j \in \aleph$;
2.2. $\alpha_i \in \{3,4\}, i \in \overline{1,k}$, moreover $\alpha_i = 4$ for all $i < k$.

In the talk we study structural, topological, metric, and fractal properties of the distribution of random variable $\xi = \Delta_{\tau_1\eta_0\eta_1\ldots\eta_k\ldots}$, where τ_1 and η_k are independent random variables with distributions: $P\{\eta_k = i\} = p_{ik}, P\{\eta_0 = i\} = p_i, i = 0,1$.

Let us consider the set $Q_3 = \{q_0, q_1, q_2 : 0 < q_i \in R, \sum_{i=0}^{2} q_i = 1\}$. Here Q_3-representation of a number is a generalization of ternary expansion [1]. We perform similar research for the random variable

$$\xi = \overline{\triangle}^{Q_3}_{\tau_1\tau_2\tau_3\ldots\tau_k\ldots} = \triangle^{Q_3}_{\underbrace{0\ldots0}_{\tau_1}\underbrace{1\ldots1}_{\tau_2}\underbrace{2\ldots2}_{\tau_3}\ldots\underbrace{0\ldots0}_{\tau_{3k-2}}\underbrace{1\ldots1}_{\tau_{3k-1}}\underbrace{2\ldots2}_{\tau_{3k}}\ldots},$$

where τ_k are independent discrete identically distributed random variables taking the values from the set $\{0,1,2,\ldots\} = N_0$.

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Dragomanov National Pedagogical University, Kyiv, Ukraine
E-mail address: irina-zamrij@yandex.ru, freeeidea@ukr.net

FRACTIONAL AND MULTIFRACTIONAL PROCESSES

APPROXIMATION OF FRACTIONAL BROWNIAN MOTION BY GAUSSIAN MARTINGALES INVOLVING INTEGRANDS REPRESENTED BY SEVERAL FUNCTIONAL CLASSES

O.L. Banna

Fractional Brownian Motion (FBM) with Hurst index $H \in (0,\,1)$ is a Gaussian process $\{B_t^H,\ t \geqslant 0\}$ with zero mean and covariance function $\mathrm{E}B_t^H B_s^H = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$, such that $B_0^H = 0$. We will only study the case when Hurst index $H \in (\frac{1}{2},\,1)$.

As it was proved in [2], the FBM $\{B_t^H,\ t \in [0,\,T]\}$ can be represented in the form $B_t^H = \int\limits_0^t z(t,\,s)dW_s$, where $\{W_t,\ t \in [0,\,T]\}$ is a Wiener process, $z(t,\,s) = \left(H - \frac{1}{2}\right) c_H s^{1/2-H} \cdot \int\limits_s^t u^{H-1/2}(u-s)^{H-3/2}du$ is a kernel, $c_H = \left(\dfrac{2H\cdot\Gamma(\frac{3}{2}-H)}{\Gamma(H+\frac{1}{2})\cdot\Gamma(2-2H)}\right)^{1/2}$ is a constant and $\Gamma(x)$, $x > 0$ is Gamma function. Suppose that $a : [0,\,T] \to \mathbb{R}$ belongs to $L_2[0,\,T]$ so that the integral $\int\limits_0^t a(s)dW_s$, $t \in [0,\,T]$ exists.

It is proved that the distance $\inf\limits_{a\in L_2[0,\,T]} \sup\limits_{0\leqslant t\leqslant T} \mathrm{E}(B_t^H - \int\limits_0^t a(s)dW_s)^2$ between fractional Brownian motion and the whole space of such integrals in non-zero [1]. This distance is estimated from below. The values of the distances between fractional Brownian motion and several subspaces of the space of Gaussian martingales are established. In some cases these distances are estimated from above and the numerical calculations are involved. As an auxiliary but interesting results, the bounds from below and from above for the coefficient appearing in the representation of fBm via Wiener process are established and some new inequalities for Gamma-functions are obtained.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: bannaya@univ.kiev.ua

APPROXIMATION OF FRACTIONAL BROWNIAN MOTION WITH GAUSSIAN MARTINGALES

Oksana Banna, Vadym Doroshenko, Yuliya Mishura, Georgiy Shevchenko, Sergiy Shklyar

Let $H \in (1/2, 1)$. Denote $\alpha = H - 1/2$. Consider the kernel

$$K(t,s) = \begin{cases} C_\alpha s^{-\alpha} \int_s^t u^\alpha (u-s)^{\alpha-1} du, & s \leqslant t \\ 0, & s > t, \end{cases}$$

where $C_\alpha = \alpha\sqrt{\frac{(2\alpha+1)\Gamma(1-\alpha)}{\Gamma(\alpha+1)\Gamma(1-2\alpha)}}$, Γ is Gamma function.

Let $\{W_t, t \geqslant 0\}$ be a Wiener process. Then $B_t^H = \int_0^t K(t,s)dW_s, t \geqslant 0$ is a fractional Brownian motion with Hurst index H ([1]).

We study the problem of approximation of $B_t^H, t \in [0,1]$ in the square-mean sense by Gaussian martingales of the form $\int_0^t x(s)dW_s, x \in L_2[0,1]$. This problem is equivalent to the problem of minimization of the functional F on $L_2[0,1]$ of the form

$$F(x) = \sup_{t\in[0,1]} \sqrt{E(B_t^H - \int_0^t x(s)dW_s)^2} = \sup_{t\in[0,1]} \sqrt{\int_0^t (K(t,s) - x(s))^2 ds}.$$

Theorem 1. *Functional F has the unique minimizing argument on $L_2[0,1]$.*

Let $a \in L_2[0,1]$ be the function on which functional F attains its minimal value.

Theorem 2. *There exists $\phi : [0,1] \to \mathbb{R}$ such that $s \leqslant \phi(s) \leqslant 1, s \in [0,1]$ and $a(s) = K(\phi(s),s), s \in [0,1]$.*

Let $x \in L_2[0,1]$. Consider the function

$$g_x(t) = \sqrt{\int_0^t (x(s) - K(t,s))^2\, ds}, t \in [0,1].$$

Theorem 3. *1) The function $g_a(t), t \in [0,1]$ attains its maximum value at $t = 1$. So, $F(a) = g_a(1)$.*

2) Let $t^* = \sup\{t \in (0,1) : g_a(t) = F(a)\}$ (set $t^* = 0$ if this set is empty). If $t^* < 1$ then $a(t) = K(1,t)$ for almost all $t \in [t^*, 1]$.

Theorem 4. *Let $x \in L_2[0,1]$. Define function $K_x(t,s)$ with the formula*

$$K_x(t,s) = \begin{cases} K(t,s) & \text{if} \quad t \geqslant s, \\ x(s) & \text{if} \quad t < s. \end{cases}$$

The function x is the minimum of the functional F if and only if there exists random variable ξ which takes values from $[0,1]$ and the following conditions hold

$$x(s) = \mathsf{E}\, K_x(\xi, s) \qquad \text{almost everywhere on } [0,1], \tag{1}$$

$$g_x(\xi) = F(x) \qquad \text{almost sure.} \tag{2}$$

The table contains results of numerical calculations of the minimal value of the functional F.

H	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
$\inf F$	0.0013	0.0051	0.0112	0.0200	0.0320	0.0482	0.0705	0.1023	0.1511

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: okskot@ukr.net, vadym.doroshenko@gmail.com, yumishura@gmail.com, zhora@univ.kiev.ua, shklyar@univ.kiev.ua

GENERALIZED FRACTIONAL CONTINUOUS TIME RANDOM WALKS

Luisa Beghin

We analyze fractional forms of generalized continuous time random walks (GCTRWs) under the assumption of i.i.d. exponential or Mittag-Leffler jumps. The process representing the number of jumps, instead of being a renewal counting process (as in the case of compound Poisson, or fractional Poisson, processes) is assumed to be either a point process with independent, but not identically distributed, waiting times, or a Markov process with more than one occurrence at a time.

The first model is obtained by subordinating the random walk by the fractional Yule process: the explicit density of the GCTRW is obtained together with the fractional differential equation satisfied by it.

In the second case, where the counting process is replaced by the fractional-difference Poisson process of order $\alpha < 1$ (defined in Orsingher and Polito (2012)), the driving equation is fractional, with respect to the time argument, but with order greater than one. Moreover, the density satisfies a fractional master equation, which generalizes the well-known Kolmogorov one.

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Department of Statistical Sciences, Sapienza University of Rome,P.le A. Moro 5, I-00185 Roma, Italy

E-mail address: luisa.beghin@uniroma1.it

AMBIT PROCESSES AND RELATED ISSUES

José Manuel Corcuera

Ambit processes are processes of the form

$$Y(x) = \int_{A(x)} g(x-\xi)\sigma(\xi)W(d\xi),$$

where $x \in R^n, g : R^n \to R$, with $g(x_1, .., x_n) = 0$ if $x_1 < 0$ (the first coordinate indicates time), σ is the intermittency or volatility parameter and $A(x)$ is the so called "ambit set". In this talk we will review some applications of ambit processes in turbulence and finance and some mathematical problems connected with them.

Facultat de Matemàtiques, Universitat de Barcelona, Gran Via de les Corts Catalanes 585, Barcelona 08007, Spain
E-mail address: jmcorcuera@ub.edu

ON THE ASYMPTOTIC PROPERTIES OF THE OREY INDEX

K. Kubilius

For real mean zero Gaussian process with stationary increments Orey [2] (see also [1]) suggested following definition of index.

Definition 1. Let X be a real-valued mean zero Gaussian stochastic process with stationary increments and continuous in quadratic mean. Let σ_X be the incremental variance of X given by $\sigma_X^2(h) = \mathbf{E}[X(t+h) - X(t)]^2$ for $t, h \geq 0$. Define

$$\widehat{\beta}_* := \inf\left\{\beta > 0 : \lim_{h\to 0} \frac{h^\beta}{\sigma_X(h)} = 0\right\} = \limsup_{h\to 0} \frac{\ln \sigma_X(h)}{\ln h} \tag{1}$$

and

$$\widehat{\beta}^* := \sup\left\{\beta > 0 : \lim_{h\to 0} \frac{h^\beta}{\sigma_X(h)} = +\infty\right\} = \liminf_{h\to 0} \frac{\ln \sigma_X(h)}{\ln h}\,. \tag{2}$$

If $\widehat{\beta}_* = \widehat{\beta}^*$ then X has the Orey index β_X.

If Gaussian process with stationary increments has Orey index then almost all sample paths satisfy a Hölder condition of order γ for each $\gamma \in (0, \beta_X)$. For fractional Brownian motion index $\beta_X = H$.

The purpose of this talk is to introduce extension of the definition of the Orey index for Gaussian processes which may not have stationary increments. Our goal is to estimate the Orey index for Gaussian process from discrete observations of its sample paths and consider the asymptotic properties of its estimator.

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VU Institute of Mathematics and Informatics, Akademijos 4, LT-08663 Vilnius, LITHUANIA
E-mail address: kestutis.kubilius@mii.vu.lt

FRACTIONAL POISSON RANDOM FIELD

N. Leonenko[1], E. Merzbach[2]

There are essentially four different approaches to the concept of Fractional Poisson process on the real line: The "integral representation" method follows the integral representation of the Fractional Brownian motion, replacing the Brownian motion by the Poisson process (Wang, Wen, Zhang). Another approach that we can call the "Renewal" approach consists of considering the characterization of the Poisson process as a sum of independent random variables, and relaxing the independent assumption (Mainardi, Gorenflo, E. Scalas). A third approach, the "differential" one, uses the differential equations of the Poisson process and replaces them by fractional derivatives (Beghin-Orsingher). Finally, using "inverse subordinator" a kind of Fractional Poisson process can be constructed (Meerschaert, Nane, Vellaisamy). Here, we will follow the fourth method to generalize and define a Fractional Poisson field parametrized by points by the Euclidean space $\mathbb{R}^d$, as has been done for fractional Brownian fields.

[1] School of Mathematics, Cardiff University, Senghennydd Road, Cardiff CF24 4AG, UK
E-mail address: leonenkon@cardiff.ac.uk
[2] Department of Mathematics, Bar-Ilan University, Ramat-Gan 52900, Israel
E-mail address: merzbach@macs.biu.ac.il

ODD-ORDER PSEUDO-PROCESSES AND STABLE LAWS

Enzo Orsingher[1], Mirko D'Ovidio[2]

We obtain generalized Cauchy r.v.'s by composing pseudo processes of odd-order with positively skewed stable random variables. We show that the distributions obtained are solutions of higher-order Laplace equations and also of second order equations. By composing the pseudo processes with iterated positively skewed stable random variables we obtain laws whose structure is studied.

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[1] Sapienza University of Rome, Faculty of Ingegneria dell'Informazione, Informatica e Statistica
E-mail address: enzo.orsingher@uniroma1.it

[2] Sapienza University of Rome, Faculty of Ingegneria Civile ed Industriale
E-mail address: mirko.dovidio@uniroma1.it

THE SPACE-FRACTIONAL POISSON PROCESS

Federico Polito

In this talk we introduce the space-fractional Poisson process. The state probabilities are governed by some difference-differential equations involving a fractional difference operator often found in the study of time series exhibiting long memory. We explicitly obtain the one-dimensional distributions and the probability generating function which is also expressed as the distribution of the minimum of i.i.d. uniform random variables. Connections with discrete stable distributions are analysed and discussed, and a useful subordination relation involving the stable subordinator is proved. The comparison with the time-fractional Poisson process is investigated and furthermore, we arrive at the more general space-time fractional Poisson process of which we give the explicit distribution. Finally we outline some possible generalisations of the time-fractional Poisson process and connect them to an integral operator with a generalised Mittag-Leffler function in the kernel.

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University of Turin, Via Carlo Alberto, 10, Turin
E-mail address: federico.polito@unito.it

SMOOTH APPROXIMATIONS FOR FRACTIONAL AND MULTIFRACTIONAL FIELDS

K.V. Ralchenko

We consider a Gaussian random field $\{B(t), t \in [0, T_1] \times [0, T_2]\}$ in the plane which is continuous almost surely and satisfies the following condition on its increments: for all $s, t \in [0, T_1] \times [0, T_2]$

$$\mathrm{E}(B(s_1, s_2) - B(s_1, t_2) - B(t_1, s_2) + B(t_1, t_2))^2 \leqslant C(|t_1 - s_1|\, |t_2 - s_2|)^\lambda,$$

where $C > 0$ and $\lambda > 1$ are some constants. This, in particular, includes anisotropic fractional and multifractional Brownian sheets.

We construct approximations for $B(t)$ in a certain Besov, or fractional Sobolev, spaces, by absolutely continuous fields. This allows one to approximate stochastic integrals with respect to fractional Brownian sheet by usual integrals and consequently, to approximate solutions of stochastic partial differential equations involving fractional noise by solutions of partial differential equations with a random source, which in many aspects are similar to non-random partial differential equations.

National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: k.ralchenko@gmail.com

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APPROXIMATION OF FRACTIONAL BROWNIAN FUNCTIONALS BY FUNCTIONALS OF INCREMENTS

T.O. Shalaiko, G.M. Shevchenko

Let $\{B_t^H, t \in [0,T]\}$ be a fractional Brownian motion with the Hurst parameter $H > 1/2$, $\xi = \xi(B_t^H, t \in [0,T])$ be a functional of it, $\pi = \{t_k = kT/N, k = 0, \ldots, N\}$ be a uniform partition of $[0,T]$.

We study a question of approximation of ξ by functionals of the increments of fBm, i.e. random variables of the form $f(\Delta B_{t_k}^H, k = 1, \ldots, N)$, where $\Delta B_{t_k}^H = B_{t_k}^H - B_{t_{k-1}}^H$, $k = 1, \ldots, N$. More specifically, we are interested in lower bounds for accuracy of such approximation. This question is motivated by analysis of accuracy of numerical schemes for stochastic differential equations driven by fractional Brownian motion.

Our main result is following.

Theorem 1. *Let ξ have a zero mean and be such that the function f_1 in its first-order term of the Itô–Wiener chaos expansion has a bounded essentially non-zero derivative, then for all $N \geqslant 1$*

$$\operatorname{var}(\xi | \Delta B_{t_1}^H, \ldots, \Delta B_{t_N}^H) \geq C N^{-4H}.$$

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Department of Probability Theory, Statistics and Actuarial Mathematics, Kyiv National Taras Shevchenko University, 64 Volodymyrska str., 01601 Kyiv, Ukraine

E-mail address: tarasenya@gmail.com, zhora@univ.kiev.ua

RECENT PROGRESS IN MIXED STOCHASTIC DIFFERENTIAL EQUATIONS

G.M. Shevchenko

The talk will be devoted mixed stochastic differential equations of the form

$$X_t = X_0 + \int_0^t a(s, X_s)\, ds + \int_0^t b(s, X_s)\, dW_s + \int_0^t c(s, X_s)\, dB_s^H$$

driven by fractional Brownian motion B^H and Wiener process W, where the integral with respect to fractional Brownian motion is the generalized Lebesgue–Stieltjes integral, and integral with respect to Wiener process is Itô integral. Under fairly mild assumptions (basically the same assumptions as for equation involving fBm only) we prove existence and uniqueness of solution for such equations. Further we study discrete (Euler) approximations for such equations and estimate the rate of convergence. Also we prove convergence of approximations of solutions to mixed equations by solutions of usual Ito stochastic equations with random drift. As a by-product, we prove a stochastic viability theorem and a comparison theorem for mixed equation.

This talk is based on joint research with Yuliya Mishura.

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Mechanics and Mathematics Faculty, Kiev National Taras Shevchenko University, 64 Volodymyrska, 01601 Kiev, Ukriane

E-mail address: zhora@univ.kiev.ua

GAUSSIAN AND RELATED PROCESSES AND RANDOM FIELDS

UPPER ESTIMATES OF PERCOLATION THRESHOLDS OF BERNOULLI'S RANDOM FIELDS ON PLANE PERIODIC GRAPHS

E.S. Antonova, Yu.P. Virchenko

The percolation problem from the fixed vertex $z \in V$ of mosaics $\langle V, \Phi \rangle$ connected with the Bernoulli random field $\tilde{c}(x)$, $x \in V$, $\Pr\{\tilde{c}(x) = 1\} = c$ is studied.

The random field $\langle \tilde{c}(x); x \in V \rangle$ has the percolation from the vertex $z \in V$ if the probability $P(c, z)$ of the event consisting of the existence of an infinite nonintersecting path with the initial vertex z does not equal to zero. The characteristics $c_* = \inf\{c : P(c, z) > 0\}$ of the percolation probability is named the percolation threshold. It does not depend on z if the mosaics represents the so-called periodic graph [1]. We propose the method of upper estimates obtaining of this characteristics.

For infinite mosaic $\langle V, \Phi \rangle$, it is possible to build the so-called adjoint graph $\langle V, \Phi^* \rangle$ whose the adjacency set Φ^* is obtained on the basis of Φ by the addition of supplementary bonds to it. In this case there is the theorem [1] which asserts that the external boundaries of finite clusters represent simple cycles on the graph being adjoint to the graph $\langle V, \Phi \rangle$.

Let $P(W)$ be the probability of the event consisting of the fact that the finite cluster W is contained in the random realization $\tilde{c}(x)$, $x \in V$ on the mosaic. The elementary estimates $P(W) < (1 - c)^{|\partial_+ W|}$ is valid. Further, let $n(s, z)$ be the number of not shortened cycles on $\langle V, \Phi^* \rangle$ surrounding the vertex z. Then, the convergence of the series

$$\sum_{s \geqslant 3} (1 - c)^s n(s, z) < \infty$$

is the sufficient condition of existence of percolation from the vertex z.

The number $n(s, z) \equiv n(s)$ does not depend on z for uniform periodic graphs. In this case the sufficient condition of percolation existence may be weakened taking into account the availability of internal boundary when the upper estimate of $P(W)$ is obtained. For example, in the case of periodic graph having the form of the so-called square lattice, we have

$$\sum_{s \geqslant 3} (1 - c)^s c^{s/3} n(s) < \infty \,. \tag{1}$$

Upper estimates of the percolation threshold are obtained on the basis of this sufficiency condition using the upper estimate of the combinatoric function $n(s)$. It is attained by the obvious estimate $n(s) < m(s)$ where $m(s)$ is the number of nonintersecting not shortened paths on $\langle V, \Phi \rangle$ from the fixed vertex $z \in V$ and the inequality $m(s_1 + s_2) < m(s_1)m(s_2)$. The computer evaluation of some first values $m(s) = 1, 2, 3, ..., l$ up to the lth one inclusively permits to write the upper estimate of the lth order for the series one (1) dividing it on finite sets containing the equal number l of subsequent terms. In a result, the series (1) convergence is followed from the convergence of the series

$$\sum_{k=1}^{\infty} \left(a^l m(l) \right)^k < \infty \,, \qquad a = (1 - c)c^{1/3} < \infty \,.$$

The estimate c_l of the percolation threshold c_* is obtained on the basis of the root of the equation $a^l m(l) = 1$ being maximal in $[0, 1]$.

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Belgorod State University, Russia
E-mail address: `virch@bsu.edu.ru`

TIME DEPENDENT RANDOM FIELDS ON SPHERICAL NON-HOMOGENEOUS SURFACES

Mirko D'Ovidio

In recent years a growing literature has been devoted to the study of the random fields on the sphere and their statistical analysis. Many researchers have focused on the construction and characterization of random field indexed by compact manifolds as the sphere $\mathbb{S}^2_r = \{\mathbf{x} \in \mathbb{R}^3 : |\mathbf{x}| = r\}$. The sphere represents an homogeneous surface on which the random field is observed. The interest in studying random fields on the sphere is especially represented by the analysis of the Cosmic Microwave Background (CMB) radiation which is currently at the core of physical and cosmological research. CMB radiation is a thermal radiation filling the observable universe almost uniformly and is well explained as radiation associated with an early stage in the development of the universe. From a mathematical viewpoint, the CMB radiation can be interpreted as a realization of an isotropic, mean-square continuous spherical random field for which a spectral representation given by means of spherical harmonics holds. Due to the Einstein cosmological principle (on sufficiently large scales the universe looks identical everywhere in space (homogeneity) and appears the same in every direction (isotropy)) the CMB radiation is an isotropic image of the early universe. Nevertheless, such a nature of the CMB radiation can be affected by anisotropies as those due to the gravitational lensing for instance.

Beside the interest on random fields, particular attention has been also paid, in last years, by yet other researchers in studying fractional diffusion equations. These equations are related to anomalous diffusions or diffusions in non-homogeneous media, with random fractal structures for instance. The solutions to fractional diffusion equations are strictly related with stable densities. Indeed, the stochastic solutions we are dealing with can be realized through subordination and therefore we obtain processes with randomly varying times which are inverse to stable subordinators.

Our interest in this work is to study the stochastic solution to a time-fractional Cauchy problem on $\mathbb{S}^2_1$ and obtain a new random structure for the sphere by means of which the random field is indexed. In such a way we want to provide a new random field in which the anisotropies of the CMB radiation can be explained.

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Dipartimento di Scienze di Base e Applicate per l'Ingegneria, Sapienza Università di Roma

E-mail address: mirko.dovidio@sbai.uniroma1.it

STATISTICAL SIMULATION OF 4D RANDOM FIELDS BY MEANS OF KOTELNIKOV-SHANNON DECOMPOSITION

Kateryna Fedorenko

The real-valued random fields $\xi(t,x), t \in (-\infty,+\infty), x \in R^3$ homogeneous on variable t and homogeneous and isotropic on variable x are studied. For such random fields with a bounded spectrum the analogue of Kotelnikov -Shannon theorem is presented. The problem of the approximation homogeneous on the variable t and homogeneous and isotropic on the variable x random fields on $R \times R^3$ by the random fields with a bounded spectrum is considered. The mean-square estimates of approximation homogeneous on time and homogeneous and isotropic on ρ,θ,φ random field with a bounded spectrum $\xi(t,\rho,\theta,\varphi)$ on $R \times R^3$ by the partial sums of rows, which are the models of such field are found. Models for such random fields are constructed. Two algorithms of statistical simulation the Gaussian random field $\xi(t,x)$ on $R \times R^3$ on the basis of Kotelnikov-Shannon decomposition are built. It is possible to generate on a computer adequate realizations of such Gaussian random field by means of these algorithms.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: slims_mentol@mail.ru

APPROXIMATION COMPLEXITY OF TENSOR PRODUCT-TYPE RANDOM FIELDS WITH HEAVY SPECTRUM

A.A. Khartov

Consider a sequence of Gaussian tensor product-type random fields, given by

$$X_d(t) = \sum_{k\in\widetilde{\mathbb{N}}^d} \prod_{l=1}^{d} \lambda_{k_l}^{1/2}\, \xi_k \prod_{l=1}^{d} \psi_{k_l}(t_l), \quad t \in [0,1]^d, \tag{1}$$

where $(\lambda_i,\psi_i)_{i\in\widetilde{\mathbb{N}}}$ are all eigenpairs ($\lambda_i > 0, i \in \widetilde{\mathbb{N}}$) of the covariance operator of the process X_1; $(\xi_k)_{k\in\widetilde{\mathbb{N}}}$ are standard Gaussian random variables. The eigenvalues $(\lambda_i)_{i\in\widetilde{\mathbb{N}}}$ satisfy the assumption $\Lambda := \sum_{i\in\widetilde{\mathbb{N}}} \lambda_i < \infty$. For any $d \in \mathbb{N}$ sample paths of X_d almost surely belong to $L_2([0,1]^d)$ supplied with a norm $\|\cdot\|_{2,d}$.

The numbers $\lambda_k := \prod_{l=1}^{d} \lambda_{k_l}$, $k \in \widetilde{\mathbb{N}}^d$ are eigenvalues of the covariance operator of X_d. We approximate X_d, $d \in \mathbb{N}$ with n-th partial sum of series (1) corresponding to n maximal eigenvalues λ_k. Let us denote this sum by $X_d^{(n)}$.

Consider the *average approximation complexity*

$$n_d^{avg}(\varepsilon) := \min\Big\{ n \in \mathbb{N} : \mathbf{E}\,\|X_d - X_d^{(n)}\|_{2,d}^2 \leqslant \varepsilon^2\, \mathbf{E}\,\|X_d\|_{2,d}^2 \Big\}$$

and the *probabilistic approximation complexity*

$$n_d^{pr}(\varepsilon,\delta) := \min\Big\{n\in\mathbb{N} : \mathbf{P}\left(\|X_d - X_d^{(n)}\|_{2,d}^2 > \varepsilon^2\,\mathbf{E}\,\|X_d\|_{2,d}^2\right) \leqslant \delta\Big\}.$$

They are investigated under the assumption $\sum_i (-\ln\lambda_i)^2\,\lambda_i < \infty$ in the paper [1]. Suppose $\sum_{i:\,\lambda_i<e^{-x}} \lambda_i \sim C\,x^{-\alpha},\ x\to+\infty$, where $\alpha\in(0,2)$ and the constant $C>0$; we prove that for any fixed $\varepsilon\in(0,1)$ and some extremely mild conditions on the level $\delta=\delta(d)$, we have $\ln\big(n_d^{avg}(\varepsilon)/n_d^{pr}(\varepsilon,\delta)\big) = o(d^{1/\alpha})$ as $d\to\infty$.

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St.Petersburg State University, Department of Mathematics and Mechanics
E-mail address: AlexeyKhartov@gmail.com

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RANDOM FIELDS: M. YADRENKO’S CONTRIBUTION AND RECENT TRENDS

Yu.V. Kozachenko[1], O.I. Klesov[2]

A survey of Yadrenko’s results in the area of random fields is given. Some recent developments of his ideas are discussed.

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[1] National Taras Shevchenko University of Kyiv
E-mail address: yvk@univ.kiev.ua

[2] National Technical University of Ukraine “KPI”
E-mail address: klesov@math.uni-paderborn.de

SPACES OF RANDOM VARIABLES $\mathbf{F}_\psi(\Omega)$

Yu.V. Kozachenko[1], Yu.Yu. Mlavets[2]

The spaces of random variables $\mathbf{F}_\psi(\Omega)$ are considered. These spaces were introduced in [1]. The properties of these spaces and the properties of random processes of these spaces are studied. Results are used to calculate integrals by Monte Carlo method with a given accuracy and reliability. In particular, the conditions under which the space $\mathbf{F}_\psi(\Omega)$ has the property $\mathbf{H}$ [2] are found and estimates of distribution of supremums of these spaces are researched.

Theorem 1 ([2]). *Let $X(t)$ – separable process on $(\mathbf{T},\rho)$ of space $\mathbf{F}_\psi(\Omega)$ and the condition holds $\sup\limits_{\rho(t,s)\le h}\big\|X(t)-X(s)\big\|_\psi \le \sigma(h)$, where $\sigma(h)$ continuous monotonically increasing function such that $\sigma(0)=0$. If, for any $\varepsilon>0$ the next condition holds $\int\limits_0^\varepsilon \varkappa\left(N\left(\sigma^{(-1)}(u)\right)\right)du < \infty$ then $\sup\limits_{t\in\mathbf{T}}|X(t)| \in \mathbf{F}_\psi(\Omega)$ and $\left\|\sup\limits_{t\in\mathbf{T}}|X(t)|\right\|_\psi \le B(p)$, where $B(p) = \inf\limits_{t\in\mathbf{T}}\|X(t)\|_\psi + \frac{1}{p(1-p)}\int\limits_0^{\gamma p}\varkappa\left(N\left(\sigma^{(-1)}(u)\right)\right)du$, $\gamma = \sigma\left(\sup\limits_{t,s\in\mathbf{T}}\rho(t,s)\right)$, $\varkappa(n)$ – majorant characteristic of $\mathbf{F}_\psi(\Omega)$. Then for any $x>0$ the inequality holds*

$$P\left\{\sup_{t\in\mathbf{T}}|X(t)| > x\right\} \le \inf_{u\ge1}\frac{B^u(p)(\psi(u))^u}{x^u}.$$

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[1] Department of Probability Theory, Statistics and Actuarial Mathematics, Faculty of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Volodymyrska Str., 64, Kyiv, 01601, Ukraine
E-mail address: ykoz@ukr.net

[2] Department of Cybernetics and Applied Mathematics, Faculty of Mathematics, Uzhgorod National University, University Str., 14, Uzghorod, 88000, Ukraine
E-mail address: yura-mlavec@ukr.net

UNIFORM CONVERGENCE OF WAVELET EXPANSIONS OF FRACTIONAL BROWNIAN MOTION

Yuriy Kozachenko[1], Andriy Olenko[2], Olga Polosmak[3]

Wavelet expansions offer significant theoretical and practical advantages over classical time or frequency domain approaches to signal analysis and processing. New results on uniform convergence in probability for general classes of wavelet expansions of Gaussian random processes are given. The results are obtained under simple conditions which can be easily verified. The conditions are less restrictive than those in the literature with similar themes. Applications of the developed technique are shown for several classes of stochastic processes. In particular, the main theorem is adjusted to the fractional Brownian motion case.

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[1] Department of Probability Theory, Statistics and Actuarial Mathematics Kyiv University, Kyiv, Ukraine
E-mail address: ykoz@ukr.net
[2] Department of Mathematics and Statistics, La Trobe University Victoria 3086, Australia
E-mail address: a.olenko@latrobe.edu.au
[3] Department of Economic Cybernetics Kyiv University, Kyiv, Ukraine
E-mail address: olgapolosmak@yandex.ru

A LIMIT THEOREM FOR GENERALIZED RANDOM FUNCTIONS

S.M. Krasnitskiy[1], O.O. Kurchenko[2]

Let $\xi(\varphi)$, $\varphi \in C_0^\infty(R^1)$ be a real generalized random process such that $\forall \varphi, \psi \in C_0^\infty$, $supp\,\varphi \cap supp\,\psi = \varnothing$ the random vector $(\xi(\varphi), \xi(\psi))$ has the property K [1]. Let $\chi_{t,h} \in C_0^\infty$ such that $\forall (t,h) \in R^1 \times (0,1) : 0 \leq \chi_{t,h} \leq 1, supp\,\chi_{t,h} = (t, t+h)$ and $\chi_{t,h}(x) = 1, x \in \left[t+h^2, t+h-h^2\right]$; $(u(n)) \subset N$ be the sequence such that $\sum_{n=1}^{\infty} n/u(n) < +\infty; \chi_{k,n} = \chi_{t,h}$ for $t = k/u(n)$, $h = 1/u(n)$. The conditions for the convergence $\sum_{k=-\infty}^{\infty} (\chi_{k,n}\xi(\varphi))^2 - \sum_{k=-\infty}^{\infty} E(\chi_{k,n}\xi(\varphi))^2 \to 0$ in mean square and with probability as $n \to \infty$ are obtained. This convergence can be considered as some analogue of the Levy-Baxter theorems for usual random functions.

Example 1. Let $\xi_c(\varphi)$, $\varphi \in C_0^\infty(R^1)$ be a real generalized random process of Gaussian white noise on R^1 having a spectral density $c = \text{const} > 0$, so [2] $E\xi(\varphi) = 0$, $r(\varphi, \psi) \equiv E\xi(\varphi)\overline{\xi(\psi)} = c\int_{-\infty}^{\infty} \tilde{\varphi}(\lambda)\overline{\tilde{\varphi}}(\lambda)d\lambda$, where $\tilde{\varphi}$ is a Fourier transform of φ. Then $\lim_{n\to\infty} \sum_{n=-\infty}^{\infty} (\chi_{k,n}\xi(\varphi))^2 = c\int_{-\infty}^{\infty} \varphi^2(t)dt$ with probability one. Probabilistic measures corresponding to processes $\xi_c(\varphi)$ and $\xi_d(\varphi)$ with $c \neq d$ are orthogonal (see also [3]).

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[1] Kyiv National University of Technology and Design, Informational Technology Department, 01601 Kyiv
E-mail address: krasnits@mail.ru
[2] National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: olkurchenko@ukr.net

ASYMPTOTICS FOR NONLINEAR FUNCTIONALS OF SPHERICAL GAUSSIAN EIGENFUNCTIONS

Domenico Marinucci[1], Igor Wigman[2]

We provide Central Limit Theorems and Stein-like bounds for the asymptotic behaviour of nonlinear functionals of spherical Gaussian eigenfunctions. Our investigation makes use of a careful analysis of higher order moments for Legendre polynomials, plus recent results on Malliavin calculus and Total Variation bounds for Gaussian subordinated fields. We discuss application to geometric functionals like the Defect and invariant statistics, e.g. polyspectra of isotropic spherical random fields. Both of these have relevance for applications, especially in an astrophysical environment.

[1] Department of Mathematics, University of Rome Tor Vergata
E-mail address: marinucc@mat.uniroma2.it
[2] Department of Mathematics, Cardiff University
E-mail address: WigmanI@cardiff.ac.uk

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PATHS IN GAUSSIAN EXCURSIONS BETWEEN POINTS

E. Moldavskaya

The structure of Gaussian random fields over high levels is a well researched and well understood area, particularly if the field is smooth. However, the question as to whether or not two or more points which lie in an excursion set belong to the same connected component has constantly eluded analysis. We study this problem from the point of view of large deviations, finding the asymptotic probabilities that two such points are connected by a path laying within the excursion set, and so belong to the same component. In addition, we obtain a characterization and descriptions of the most likely paths, given that one exists.

This is joint results with Robert Adler and Gennady Samorodnitsky.

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Technion, Haifa, Israel, 32000
E-mail address: elinamoldavskaya@gmail.com

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DISCRETE REPRESENTATION OF RANDOM FUNCTION IN NORMED SPACES

O.I. Ponomarenko

The representations for generalized vector-valued random function of second order over set T in the form of infinite or finite sums with members which are products of scalar functions and random coefficient under wide assumption with respect of T and properties of random function are considered.

In the first part of this communication we study such representations in the case of generalized random function with values in normed space X under assumptions that their covariance functions are products of positive definite kernel over compact topological space T or measurable space T with positive measure and some positive operator from dual space X' for X into second dual space X". The methods for obtaining discrete representations use spectral theory for general integral operators of Hilbert-Shmidt and general representation theory for vector-valued random functions [1].

The second part deals with applications of abstract harmonical analysis to different classes of invariant generalized random function in X over compact homogeneous space Q and compact topological groups [2]. In the case of arbitrary T we use the techniques of different factorization representations for covariance functions of generalized random functions over T [3].

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: dmponoma@mts.com.ua

A MODEL CONSTRUCTION OF STOCHASTIC PROCESS TAKING INTO ACCOUNT THE PROCESS DERIVATIVE

I. Rozora

Consider a stochastic process $X = \{X(t), t \in \mathbf{T}\}$ which correlation function $R(t,s)$ can be represented as

$$R(t,s) = \int\limits_{\Lambda} f(t,\lambda)\overline{f(s,\lambda)}d\mu(\lambda), \tag{1}$$

where $f(t,\cdot) \in L_2(\Lambda,\mu)$ for $t \in \mathbf{T}$.

Following result holds true.

Theorem 1. *Let $X = \{X(t), t \in \mathbf{T}\}$ be a centered second-order stochastic process with correlation function $R(t,s)$ which admits representation* (1)*, $\{g_k(\lambda), k \in \mathbb{Z}\}$ be an orthonormal basis in $L_2(\Lambda,\mu)$. Then the process $X(t)$ can be represented as a mean square convergent series*

$$X(t) = \sum_{k\in\mathbb{Z}} a_k(t)\xi_k, \tag{2}$$

where

$$a_k(t) = \int\limits_{\Lambda} f(t,\lambda)\overline{g_k(\lambda)}d\mu(\lambda), \quad \mathbf{E}\xi_k = 0, \quad \mathbf{E}\xi_k\overline{\xi_l} = \delta_{kl}.$$

Some theorems concerning conditions of uniform convergence of series (2) have been obtained.

The expansion (2) can be used for model construction of stochastic processes in different functional space with given accuracy and reliability. The conditions are found under which the model approximates stochastic process taking into account the derivative of the process.

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National Taras Shevchenko University of Kyiv, The Faculty of Cybernetics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: irozora@bigmir.net

GENERALIZED GAUSSIAN BRIDGES OF PREDICTION-INVERTIBLE PROCESSES

Tommi Sottinen, Adil Yazigi

A generalized bridge is the law of a stochastic process that is conditioned on multiple linear functionals of its path. We consider two types of representations of such bridges: orthogonal and canonical. In the orthogonal representation the bridge is constructed from the entire path of the underlying process. The orthogonal representation is given for any continuous Gaussian process. In the canonical representation the linear spaces $\mathcal{L}_t(X) = \overline{\mathrm{span}}\,\{X_s; s \leqslant t\}$ coinside for all $t < T$ for both the original process and its bridge representation. However, the canonical representation is given only for so-called prediction-invertible Gaussian processes:

A Gaussian process $X = (X_t)_{t\in[0,T]}$ is *prediction-invertible* if it can be recovered (in law, at least) from its prediction martingale:

$$X_t = \int_0^t p_T^{-1}(t,s)\,\mathrm{dE}[X_T|\mathcal{F}_s^X].$$

In discrete time all non-degenerate Gaussian processes are prediction-invertible. In continuous time this is most probably not true.

The work combines and extends the results of [1] and [2].

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University of Vaasa, Faculty of Technology, Department of Mathematics and Statistics, P.O.Box 700, FIN-65101 Vaasa, FINLAND

E-mail address: tommi.sottinen@uwasa.fi, adil.yazigi@uwasa.fi

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SOME PROBLEMS OF SPATIAL TIME PROCESSES

Gy. Terdik

One of the classical differences between classical time series and spatial processes is that in time series (defined on the real line) one can define directionality i.e. past, present and future and this is not that obvious in general space. This lack of directionality is a serious problem and a stumbling block in modelling spatial processes.

In this talk some statistical problems of nonGaussian isotropic spatial-time processes on the sphere will be considered.

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U of Debrecen, Hungaryr
E-mail address: `terdik.gyorgy@inf.unideb.hu`

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WAVELET-BASED SIMULATION OF STOCHASTIC PROCESSES WITH GIVEN ACCURACY AND RELIABILITY

I. Turchyn

Let ϕ be a father wavelet, ψ — the corresponding mother wavelet. We consider a centered second-order random process $X = \{X(t),\, t \in \mathbb{R}\}$ which correlation function can be represented as $R(t,s) = \int_{\mathbb{R}} u(t,y)\,\overline{u(s,y)}\,dy$. Such a process can be expanded in the mean square convergent series

$$X(t) = \sum_{k\in\mathbb{Z}} \xi_{0k} a_{0k}(t) + \sum_{j=0}^{\infty} \sum_{k\in\mathbb{Z}} \eta_{jk} b_{jk}(t),$$

where

$$a_{0k}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y)\overline{\hat{\phi}_{0k}(y)}dy, \quad b_{jk}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y)\overline{\hat{\psi}_{jk}(y)}dy,$$

the random variables ξ_{0k}, η_{jk} are centered and uncorrelated.

The process

$$\hat{X}(t) = \sum_{k=-(N_0-1)}^{N_0-1} \xi_{0k} a_{0k}(t) + \sum_{j=0}^{N-1} \sum_{k=-(M_j-1)}^{M_j-1} \eta_{jk} b_{jk}(t)$$

is called a model of the process $X(t)$. We say that the model $\hat{X}(t)$ approximates the process $X(t)$ with given accuracy ε and reliability $1-\delta$ $(0<\delta<1)$ in $L_p([0,T])$ if

$$\mathrm{P}\left\{ \left(\int_0^T |X(t) - \hat{X}(t)|^p dt \right)^{1/p} > \varepsilon \right\} \le \delta.$$

There have been obtained sufficient conditions for approximation of a random process by the model $\hat{X}(t)$ with given accuracy and reliability in $L_p([0,T])$ for φ-sub-Gaussian, strictly sub-Gaussian and Gaussian random processes.

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Oles Honchar Dnipropetrovsk National University, Gagarin av. 72, Dnipropetrovsk, Ukraine
E-mail address: `evgturchyn@ukr.net`

SOME FUNCTIONALS OF φ-SUB-GAUSSIAN STORAGE PROCESSES

R.E. Yamnenko

Tail estimates of some functionals on random process $\{X(t), t \in T\}$ from class $V(\varphi,\psi)$ are presented. Particularly, an estimate of probability like

$$\mathsf{P}\left\{ \sup_{s\leqslant t; s,t\in B} (X(t) - X(s) - (f(t) - f(s))) > x \right\}$$

is obtained, where $f(t)$ is a continuous function, which can be considered as the service output rate of the queue formed by the process $X(t)$. Such expression can be considered as estimate of buffer overflow probability with finite buffer size $x > 0$.

It should be noted that obtained results for random processes from the class $V(\varphi, \psi)$ also take place for Gaussian processes. As an example we consider generalized fractional Brownian motion defined on a finite interval.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: yamnenko@univ.kiev.ua

SIMULATION OF THE STATIONARY RANDOM SEQUENCES

O.I. Vasylyk, T.O. Yanevych

We consider random sequence $\{\theta_n\}_{n\in\mathbb{Z}}$ which is stationary in wide sense, that is $E\theta_n = 0$ and $E\theta_n\theta_k = B(k-n)$, $n, k \in \mathbb{Z}$. It is assumed that it can be presented as a moving summation process $\theta_n = \sum_{k=0}^{\infty} a_k\eta_{n-k}$, where $\sum_{k=0}^{\infty} |a_k|^2 < \infty$ and the random variables $\{\eta_i\}_{i=-\infty}^{\infty}$ are centered and uncorrelated: $E\eta_i = 0$, $E\eta_i\eta_j = \delta_{ij}$, δ_{ij} is the Kronecker delta. A model for its simulation is $\widehat{\theta}_n^N = \sum_{k=0}^{N} a_k\eta_{n-k}$. The accuracy of the model is measured by $\Delta_n^N = |\theta_n - \widehat{\theta}_n^N| = \sum_{k=N+1}^{\infty} a_k\eta_{n-k}$.

Definition 1. It is said that the model $\widehat{\theta}_n^N$ simulates the sequence θ_n, $n = 1, ..., m$ with given accuracy $\varepsilon > 0$ and reliability $1-\gamma$, $(0 < \gamma < 1)$ if the following inequality holds true

$$P\{\max_{n=\overline{1,m}} |\theta_n - \widehat{\theta}_n^N| > \varepsilon\} < \gamma.$$

So, let the random variables $\{\eta_k\}$ belong to the space $Sub_\varphi(\Omega)$ (See [1]). We denote $\delta_N := \max_{n=\overline{1,m}} \tau_\varphi(\Delta_n^N)$. The main result is:

Theorem 2. *The model $\widehat{\theta}_n^N$, $n = 1, ..., m$ simulates the sequence θ_n, $n = 1, ..., m$ with given accuracy ε and reliability $1-\gamma$ if*

(i) $\delta_N \le \frac{\varepsilon}{b\varphi^{*(-1)}(M\ln m)\varphi^{*(-1)}\left(-\ln\left(\frac{\gamma m^{M-1}(b-1)}{b+1}\right)\right)}$ *and* $e^{\frac{\varphi^*(2)}{M}} \le m < \left(\frac{e^{-\varphi^*(2)}}{\gamma}\cdot\frac{b+1}{b-1}\right)^{\frac{1}{M-1}}$;

(ii) $\delta_N \le \frac{\varepsilon}{2b\varphi^{*(-1)}(M\ln m)}$ *and* $m \ge \left(\frac{e^{-\varphi^*(2)}}{\gamma}\cdot\frac{b+1}{b-1}\right)^{\frac{1}{M-1}}$,

where $M > 1$, $b > 1$ are some fixed parameters.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: ovasylyk@univ.kiev.ua, yata452@univ.kiev.ua

PEACOCS AND ASSOCIATED MARTINGALES, WITH EXAMPLES

Marc Yor

A peacock is a family of marginals which is increasing in the convex order. To such a family one can always associate a martingale with these marginals (Kellerer's theorem). I shall discuss several interesting examples.

Laboratoire de Probabilites et Modeles Aleatoires, Universite Paris VI et VII, 4 Place Jussieu - Case 188, F-75252 Paris Cedex 05; Institut Universitaire de France

E-mail address: deaproba@proba.jussieu.fr

CONVERGENCE OF GENERALIZED SPITZER'S SERIES

Y. Gregul

Let $\{X_k, k \geqslant 1\}$ be independent, identically distributed random variables, and let $S_1 = X_1, S_n = X_1 + \ldots + X_n,\ n > 1$ denote their partial sums.

Spitzer's series [1] is defined as

$$f(\epsilon) = \sum_{n=1}^{\infty} \frac{1}{n} P(|S_n| \geqslant n\epsilon). \tag{1}$$

We consider the so-called empirical analogue of this series

$$\eta_1(\epsilon) = \sum_{n=1}^{\infty} \frac{1}{n} \mathbb{I}_{\{\omega: |S_n| \geqslant n\epsilon\}}, \tag{2}$$

where $\mathbb{I}_A$ is the indicator function of a random event A. Then $E[\eta_1(\epsilon)] = f(\epsilon)$ and thus the convergence of Spitzer's series (1) is equivalent to the question about existence of the first moment of the random variables $\eta_1(\epsilon)$.

The following theorem gives conditions, when the moments of generalized Spitzer's series

$$\eta_L^{'}(\epsilon) = \sum_{n=1}^{\infty} \frac{L(n)}{n} \mathbb{I}_{\{\omega: |S_n - med(S_n)| \geqslant n\epsilon\}},$$

$$\eta_L^{''}(\epsilon) = \sum_{n=1}^{\infty} \frac{L(n)}{n} \mathbb{I}_{\{\omega: |S_n| \geqslant n\epsilon\}}$$

exist, where L is a slowly varying function.

Theorem 1. *If $E[|X|L(|X|)] < \infty$, then $E[\eta_L^{'}(\epsilon)]^2 < \infty$, and if, additionally, $E[X] = 0$, then $E[\eta_L^{''}(\epsilon)]^2 < \infty$ for all $\epsilon > 0$.*

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National Technical University of Ukraine "Kiev Polytechnical Institute"
E-mail address: greguly@mail.ru

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STRONG LAW OF LARGE NUMBERS FOR RANDOM VARIABLES WITH SUPERADDITIVE MOMENT FUNCTION

T.M. Grozian

Definition 1. A sequence of random variables $\{X_n, n \geq 1\}$ is said to have the rth ($r > 0$) moment function of superadditive structure if there exists a nonnegative function $g(i,j)$ of two arguments suth that for all $b \geq 0$ and $1 \leq k < b + l$

$$g(b,k) + g(b+k,l) \leq g(b,k+l)$$

and for some $\alpha > 1$

$$E|S_{b,n}|^r \leq g^{\alpha}(b,n),$$

where $S_{b,n} = \sum_{v=b+1}^{b+n} X_v$.

Theorem 2 (I. Fazekas and O. Klesov)**.** *Assume that a sequence of random variables $\{X_n, n \geq 1\}$ has an rth moment function of superadditive structure with $r > 0$, $\alpha > 1$. Let $q > 0$, $g_n = g(0,n)$. If $b_n = n^{1/q}$ and*

$$\sum_{n=1}^{\infty} \frac{g_n^{\alpha}}{n b_n^r} < \infty,$$

then

$$\lim_{n\to\infty} \frac{S_n}{n^{\frac{1}{q}}} = 0 \quad a.s.$$

In this work SLLN for random variables with an rth moment function of superadditive structure under normalization $b_n = n^{1/q} L(n)$ is obtained, where L is a slowly varying function.

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National Technical University of Ukraine "Kiev Polytechnical Institute"
E-mail address: tgrozian@gmail.com

ON THE NUMBER OF EMPTY BOXES IN THE BERNOULLI SIEVE

Alexander Iksanov

Let $(T_k)_{k\in\mathbb{N}_0}$ be a multiplicative random walk defined by

$$T_0 := 1, \quad T_k := \prod_{i=1}^{k} W_i, \quad k \in \mathbb{N},$$

where $(W_k)_{k\in\mathbb{N}}$ are independent copies of a random variable W taking values in $(0, 1)$. Let $(U_k)_{k\in\mathbb{N}}$ be independent random variables with the uniform $[0, 1]$ law which are independent of the multiplicative random walk. The *Bernoulli sieve* is a random occupancy scheme in which 'balls' U_k's are allocated over infinitely many 'boxes' $(T_k, T_{k-1}]$, $k \in \mathbb{N}$. Assuming that the number of balls equals n denote by L_n the number of empty boxes within the occupancy range.

Depending on the behavior of the law of W near the endpoints 0 and 1 the number of empty boxes can exhibit quite a wide range of different asymptotics.

Case $\mu < \infty$ and $\nu < \infty$: L_n converges in distribution along with all moments to some L with proper and nondegenerate law.

Case $\mu = \infty$ and $\nu < \infty$: L_n converges to zero in probability.

Case $\mu < \infty$ and $\nu = \infty$: There are several possible modes of the weak convergence of L_n, properly normalized and centered.

Case $\mu = \infty$ and $\nu = \infty$: The asymptotics of L_n is determined by the behavior of the ratio $\mathbb{P}\{W \le x\}/\mathbb{P}\{1-W \le x\}$, as $x \downarrow 0$. When the law of W assigns much more mass to the neighborhood of 1 than to that of 0 equivalently the ratio goes to 0, L_n becomes asymptotically large. In this situation the weak convergence of L_n, properly normalized without centering, takes place under a condition of regular variation. If the roles of 0 and 1 are interchanged L_n converges to zero in probability. When the tails are comparable L_n weakly converges to a geometric distribution.

National Taras Shevchenko University of Kyiv, The Faculty of Cybernetics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: iksan@univ.kiev.ua

ON A CONJECTURE OF ERDÖS ABOUT ADDITIVE FUNCTIONS

Karl-Heinz Indlekofer

For a real-valued additive function $f : \mathbb{N} \to \mathbb{R}$ and for each $n \in \mathbb{N}$ we define a distribution function

$$F_n(x) := \frac{1}{n}\#\{m \le n : f(m) \le x\}.$$

In this talk we prove a conjecture of Erdös, which asserts that in order for the sequence F_n to be (weakly) convergent, it is sufficient that there exist two numbers $a < b$ such that $\lim_{n\to\infty}(F_n(b) - F_n(a))$ exists and is positive.

The proof is based upon the use of the Stone-Čech compactification $\beta\mathbb{N}$ of $\mathbb{N}$ to mimic the behaviour of an additive function as a sum of independent random variables.

Faculty of Computer Science, Electrical Engineering and Mathematics, Department of Mathematics, University of Paderborn, Warburger Strasse 100, 33098 Paderborn, Germany
E-mail address: k-heinz@math.upb.de

THE ENHANCED STABILITY ESTIMATE OF A NONHOMOGENEOUS PERTURBATION OF THE RENEWAL EQUATION ON HALFLINE

N.V. Kartashov

Let G is a distribution on $\mathfrak{B}(\mathbb{R}_+)$ with mean m_G, such that some convolution G^{*m} have a absolutely continuous component. C. Stone [1] obtains for the renewal measure $U(B) = \sum_{n\ge0} G^{*n}(B)$ the representation $U = m_G^{-1}L+V$, where L is the Lebesque measure, and V is a signed bounded measure under assumption of finite second moment of G. Define by B_0 the class of Borel locally bounded functions on $\mathbb{R}_+$, and the class $L_1^0 \equiv \left\{y \in B_0 : \lim_{t\to\infty} y(t) = 0,\ \int_{[0,\infty)} |y(s)|\, ds < \infty\right\}$.

Consider the inhomogeneous perturbation of the renewal equation induced by G for a function $x \in B_0$ as the Volterra equation

$$x(t) = y(t) + \int_0^t x(t-s)(1+\gamma(t,s))G(ds),\ t \in \mathbb{R}_+,\ (1)$$

where γ is the (small) local bounded density of of a absolute continuous perturbation of the distribution G. Let also $x_0 \in B_0$ is the solution of (1) for $\gamma \equiv 0$.

Define following (small) parameters of the perturbation.

$$\lambda_t(s) \equiv m^{-1}\int_0^{t-s}\gamma(s+u,u)G(du), 0 \le s \le t,\ \lambda(s) \ge \max(\sup_{t\ge s}(\lambda_t(s))^{\pm}),\ s \ge 0,\ \Lambda(t) \equiv \int_0^t \lambda(s)ds,$$
$$\delta(t) = \int_0^t |\gamma(t,s)|\, G(ds),\quad \delta_V(t) \equiv |V| * \delta(t),\quad \varepsilon_V \equiv \sup_{t\ge0}\delta_V(t),$$

$$\varepsilon_\Lambda^\pm \equiv \int_0^\infty \delta_V(s)\sup_{t\ge s}(\lambda_t(s))^\pm ds, \quad \varepsilon_\Lambda = \max(\varepsilon_\Lambda^+, \varepsilon_\Lambda^-),$$

One of results [2] states that under conditions $\varepsilon_V + \varepsilon_\Lambda < 1$ *and* $y \in L_1^0$ the equations (1) have the unique solutions $x_0,\ x \in B_0$, and

$$\sup_{t\ge 0}\exp(-\Lambda(t))\,|x(t)| \le (1-\varepsilon_V-\varepsilon_\Lambda)^{-1}\sup_{t\ge 0}|x_0(t)| < \infty,$$
$$\sup_{t\ge 0}\exp(-\Lambda(t))\,|x(t)-x_0(t)| \le \varepsilon_V\sup_{t\ge 0}|x(t)| + \int_0^\infty |x_0(s)|\exp(-\Lambda(s))d\Lambda(s).$$

Under conditions $\gamma(t,s)\ge -1$ and $\delta \in L_1^0$ there exists the limit $\lim_{t\to\infty} x(t)$.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: nkartashov@skif.com.ua

SOME GENERALIZATIONS OF DOOB'S TRANSFORMATION THEOREM

N.V. Kruglova

Let $Y(t)$ be a zero mean Gaussian process with such covariance function: $E[Y(s)Y(t)] = u(s)v(t), \quad s \le t$. We generalizate Doob's Transformation Theorem for such process. We get a criterion of equivalence between Gaussian process and Wiener process.

Theorem 1. *Let $Y(t)$ be a Gaussian process with $E[Y(t)] = 0$ and covariance function $E[Y(s)Y(t)] = u(s)v(t), \quad s \le t$. Assume that $\varphi(t)$ is continuous and strictly increasing function, η is continuous function. Then $\frac{Y(\varphi(t))}{\eta(t)}$ and $w(t)$ are stochastically equivalent processes if and only if*

$$\varphi(t) = \left(\frac{u}{v}\right)^{-1}(c^2 t),$$
$$\eta(t) = cv\left(\left(\frac{u}{v}\right)^{-1}(c^2 t)\right),$$

where $c \ne 0$.

Theorem 2. *Let $Y(t)$ be a Gaussian process with $E[Y(t)] = 0$ and $E[Y(s)Y(t)] = u(s)v(t), \quad s \le t$. Assume that $\varphi(t)$ is continuous and strictly increasing function, η is continuous function. Then $\frac{Y(\varphi(t))}{\eta(t)}$ and Brownian bridge are stochastically equivalent processes if and only if*

$$\varphi(t) = \left(\frac{u}{c^2 v + u}\right)^{-1}(t),$$
$$\eta(t) = \frac{(c^2 v + u)\left(\left(\frac{u}{c^2 v+u}\right)^{-1}(t)\right)}{c},$$

where $c \ne 0$.

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National Technical University of Ukraine "Kiev Polytechnical Institute"

E-mail address: natahak@ukr.net

RECORDS AND RANDOM FIELDS

Allan Gut (Uppsala) and Ulrich Stadtmüller*(Ulm)[1]

We consider an i.i.d. random field $(X_{\mathbf{k}})_{\mathbf{k}\in\mathbf{N}^d}$ along with an increasing sequence of nested sets in $\mathbf{N}^d$

$$S_j = \bigcup_{k=1}^{j} H_k, \quad j \ge 1,$$

where e.g the sets H_j could be hyberbolas $\{\mathbf{k} \in \mathbf{N}^d \ : \ |\mathbf{k}| = j\}$. We are interested in the record times along the hyberbolas, the associated counting process and the number of records in H_j in comparison to S_{j-1}. We will give various limit theorems under mild conditions on the size of the sets H_j.

[1] University of Ulm, Institute of Number Theory and Probability Theory, 89069, Ulm, Germany

E-mail address: ulrich.stadtmueller@uni-ulm.de

UNBOUNDEDNESS OF SOLUTIONS OF SDE WITH TIME-DEPENDENT COEFFICIENTS

O.A. Tymoshenko

Consider a stochastic differential equation (SDE) with time dependent coefficient of drift and diffusion

$$d\eta(t) = g\left(\eta(t)\right)\varphi(t)dt + \sigma\left(\eta(t)\right)\theta(t)dw(t),\ \ t \geqslant 0; \eta(0) = b, b > 0. \tag{1}$$

where w is a standard Wiener process; b is a non-random positive constant; θ and φ are continuous functions, g and σ are positive continuous functions such that (1) has a continuous solution η.

Conditions under which the exact order of growth of solution η is determined almost surely (a.s) by the solution μ of the corresponding to (1) ordinary differential equation

$$d\mu(t) = g\left(\mu(t)\right)\varphi(t)dt,\ \ t \geqslant 0; \mu(0) = b, b > 0,$$

are obtained in [2].

One of the basic assumptions of [2] is that the solution of SDE

$$\lim_{t\to\infty} \eta(t) = \infty \text{ a.s.},$$

so it is natural to investigate when it holds.

Unboundedness of solution of autonomous SDE is considered in book of I.I. Gihman and A.V. Skorohod [2]. Sufficient conditions of unboundedness of solution of SDE (1) are presented in the talk.

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National Technical University of Ukraine "Kiev Polytechnical Institute"

E-mail address: `zorot@ukr.net`

LEARNING OF SYSTEMATIC LINEAR CODES FROM SAMPLES OF NOISY CODEWORDS

A.N. Alekseychuk[1], A.Yu. Gryaznukhin[2]

Let C be an unknown binary linear code with the generator matrix $G = (I_k, X)$, where I_k is the $k \times k$ identity matrix and X is a $k \times (n-k)$ matrix over the field $\mathbf{GF}(2)$. A sequence $Y_i = U_i G \oplus \eta_i$, $i = \overline{1, m}$, is observed, where $U_1, ..., U_m$ are independent equiprobable random Boolean vectors of size k, $\eta_i = (\eta_{i,1}, ..., \eta_{i,n})$ are random vectors with mutually independent coordinates that are not depended on $U_1, ..., U_m$ and are distributed as follows:

$$\mathbf{P}\{\eta_{i,s} = 0\} = 1 - \mathbf{P}\{\eta_{i,s} = 1\} = 1/2 \cdot (1 + \theta_{i,s}),$$

where $\theta_{i,s} \geq \theta > 0$ for all $i = \overline{1, m}$, $s = \overline{1, n}$. The problem is to recover X from $Y_1, ..., Y_m$, n, k, and θ.

We show that this problem reduces to solving of $n-k$ systems of linear Boolean equations with noised right-hand side and the same coefficient matrix of size $m \times k$, where the noise level depends on the maximum value ρ of the number of ones in columns of the matrix X. In particular, for any $\delta \in (0, 1)$ and

$$m = \left\lceil 8\theta^{-2(1+\rho)} \ln \left((n-k)\delta^{-1} \sum_{l=1}^{\rho} \binom{k}{l} \right) \right\rceil$$

the matrix X can be recovered from the sequence $Y_1, ..., Y_m$ with probability at least $1-\delta$ in time $O\left(m(n-k) \sum_{l=1}^{\rho} \binom{k}{l} \right)$. We also discuss the applicability of some known algorithms to the problem of recovering the matrix X and demonstrate the results of computer simulations.

Institute of special communication and information security, National Technical University of Ukraine "KPI", Moskovska st. 45/1, Kiev, Ukraine

E-mail address: [1] alex-crypto@mail.ru, [2] X67mail@gmail.com

FAST ALGORITHM FOR RECONSTRUCTION OF HIGH-PROBABLE LOW-DIMENSIONAL APPROXIMATIONS FOR BOOLEAN FUNCTIONS

A.N. Alekseychuk, S.N. Konyushok

A Boolean function $g : \{0,1\}^n \to \{0,1\}$ is said to be k-dimensional [1], $1 \leqslant k \leqslant n-1$, if it can be represented in the form $g(x) = \phi(xA)$, $x \in \{0,1\}^n$, where $\phi : \{0,1\}^k \to \{0,1\}$ and A is an $n \times k$-matrix over the field $\mathbf{GF}(2)$. It is known that (for temperate values of k) the closeness of a Boolean function to the set of k-dimensional functions is a cryptographic weakness that implies the possibility of some attacks on keystream generators based on such functions.

Let f be a Boolean function in n variables and g be a k-dimensional function such that $\mathbf{P}_X\{f(X) \neq g(X)\} \leq 2^{-(k+1)}(1-\varepsilon)$, where $\varepsilon \in (0,\ 1)$ and X is an equiprobable random Boolean vector. In [1], a probabilistic algorithm, which recovers the function g from the given f, k, and ε with probability at least $1-\delta$ in $O(2^{4k} n^2 \varepsilon^{-2} \log(2^{2k} n \delta^{-1}))$ bit operation, is proposed. We propose another algorithm with the time complexity

$$O(2^{2k} k^{-2} n^3 \varepsilon^{-2} \delta^{-1} \log(2^{2k} k^{-1} n \delta^{-1} \varepsilon^{-1})).$$

This algorithm is more efficient for not to small values of k, in particular, for $k = C \log n$, where $C > 1$.

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Institute of special communication and information security, National Technical University of Ukraine "KPI", Moskovska st. 45/1, Kiev, Ukraine

E-mail address: [1] alex-crypto@mail.ru, [2] 3tooth@mail.ru

ANALYSIS OF POSTQUANTUM ONE-WAY FUNCTIONS BASED ON COMMUTATIVE AND LOCALLY COMMUTATIVE MAPPINGS

A.V. Fesenko

The most widely used in the public key cryptography difficult problems, factorization and discrete logarithm, can be solved in polynomial time on an expected quantum computer. Though there are no high-capacity implementations of quantum computer at the moment, but their development has made a big progress. Therefore cryptographers are looking for some new problems which will be really hard in both computational models.

In this paper the hardness analysis in the quantum computational model was done for a class of commutative and locally commutative mappings with a form $E : K \times X \to X$. One of results is formulation of two problems: hidden impact on the G-torsor over an abelian group and hidden element of G-torsor over an abelian group. Partial or complete solutions of these problems have an impact on the existence of postquantum one-way functions of the reffered form. Also was showed an effective reduction of the proposed tasks to the hidden shift problem, one of the most studied problems in quantum computing model. The hidden shift problem has the subexponential solution in general and under some additional conditions it also has a number of polynomial solutions. Moreover, it was shown that the formulated problems are really simpler than the hidden shift problem, i.e. have a greater chance for existence of effective solutions even in general. However solutions for hidden shift problem as well as additional solutions for that particular problems allow to create a list of criteria under which in the quantum computing model will exist a polynomial algorithm for the inversion of commutative and locally commutative mappings. Using the proposed methods of analysis as well as a list of criteria was analyzed security of some symmetrical commutative cipher regarding known-plaintext attack. On the other hand, the proposed methods can be used for analysis of any symmetric cipher and even for some protocols such as Diffie-Hellman protocol.

National Technical University of Ukraine "Kyiv Polytechnic Institute"
E-mail address: andrey.fesenko@gmail.com

PROBABILITIES ON ALGEBRAIC GROUPS

Nikolaj Glazunov

Books by Gnedenko B.V. and Kolmogorov A.N. [3], by Gnedenko B.V. [2] and by Gnedenko B.V. with his colleagues [3, 4] contain basic facts on probability theory. On the base on the results we investigate probabilities on algebraic groups. These include probabilities on linear algebraic groups and on group Calabi-Yau (CY) manifolds.

By CY variety we understand algebraic variety X over complex numbers with zero canonical class K_X. The one-dimensional CY varieties are elliptic curves and two-dimensional CY varieties are two-dimensional abelian varieties and $K3$ surfaces.

Some recent developments led to non-commutative probabilities [1]. Non-commutative probabilities connect with non-commutative spaces and with non-commutative ring of functions on the spaces. Commutative and non-commutative probabilities are considered. The commutative case relates to one-dimensional algebraic groups and to abelian varieties [6]. The non-commutative case relates to non-commutative linear algebraic groups of dimension greater than one [7] and to non-commutative nonlinear algebraic groups. Applications to compressed sensing and computer security will be done [7, 8].

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National Aviation University, pr. Kosmonavta Komarova, 1, Kiev, 03680
E-mail address: glanm@yahoo.com

APPROACHES TO EXECUTION OF SETS OF TASKS WITH RANDOM PROCESSING TIME IN COHERENT COMPUTATIONAL SYSTEMS

P.E. Golosov[1], A.F. Ronzhin[2]

Article describes the comparison of parallel and consequent approaches of executing a group of $n \geqslant 2$ tasks with random processing time $X_1, ..., X_n$ using a parallel computational resource. Main definitions are described in [1]. Conditions for the distribution of tasks are discussed in [2].

For any plan π and $x > 0$ we denote by $t_\pi(x)$ the conditional expectation value of the time that task (indexed with k) is present in the system (being processed), under condition that its length equals x: $t_\pi(x) = M(t_{x_k}^{(\pi)} | X_k = x)$.

The class of execution plans that support the SJF (Shortest Job First) strategy we denote with P.

Theorem 1. *For any plan π from the class P is true:*

$$Mt_\pi(x) \geqslant x + (n-1)\int_0^x y dF(y).$$

For the parallel plan: $Mt_{parr}(x) = x + (n-1)\int_0^x (1 - F(y))dy$.

For the consequent plan: $Mt_{seq}(x) = x + \frac{(n-1)}{2} M\xi$.

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Institution of Russian Academy of Sciences Dorodnicyn Computing Centre Effective Algorithms Sector, Vavilova st. 40, 119333 Moscow, Russia

E-mail address: [1] `pgolosov@gmail.com`, [2] `raf-zao-zt@yandex.ru`

UPPER BOUNDS OF THE AVERAGE INTEGER DIFFERENTIAL PROBABILITIES FOR ROUND FUNCTIONS OF SPECIAL STRUCTURE

L.V. Kovalchuk, N.V. Kuchinska

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To construct estimates of the block cipher resistance to differential cryptanalysis and its various modifications, as a rule, it is necessary to estimate above the average probability of the round differential. Round function of most modern block ciphers include the composition of key adder, substitution block and permutation operator, which is linear over the field F_2 or it's extension. Therefore, the problem of block ciphers resistance estimation is reduced to the problem of constructing upper bounds of the average probabilities of such compositions or contains it as a subtask. Due to the increasing demands for the speed of block cipher algorithms, many of today's algorithms are byte-oriented, so the actual problem is to estimate the integer differential probability for the composition of a key adder, substitution block and a permutation operator, including the byte shift operator. The authors have solved the problem of constructing upper bounds for the average integer differential probability of round functions which are the compositions of a key adder, a substitution block, and a cyclic shift operator, when the shift is proportional to the length of the s-block input. In the course of solving this problem is also shown that the upper estimates of the integer differential probability will decrease if the value of cyclic shift is proportional to the length of the s-block input, so that modification described leads not only to the increase of the speed of encryption, but also to the increase of security of the algorithm against to integer differential cryptanalysis.

National Technical University of Ukraine "Kiev Polytechnical Institute"

E-mail address: `lv_kov_crypto@mail.ru, n.kuchinska@gmail.com`

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ON A SHARED SECRET KEY DISTRIBUTION IN DIFFIE-HELLMAN KEY EXCHANGE SCHEME

Machulenko O.M., Savchuk M.M., Zavadskaya L.O.

Consider the following problem. Let p be a large prime number, and α be a primitive root modulo p. Suppose k_1 and k_2 are chosen independently and uniformly in the interval $[1, p-1]$. Find the distribution of $k = \alpha^{k_1 k_2} mod p$. It is easy to see that this problem is none other than the task of finding the distribution of shared secret key in Diffie-Hellman key exchange scheme [1]. To consider the private keys k_1 and k_2 in this scheme as uniformly distributed is quite natural. However, the question of the distribution of the secret key p has been neglected. In the present work is shown that the distribution of the secret key k is not uniform for any $p > 2$ and a formula for this distribution based on the decomposition of the number $p-1$ is obtained.

Theoretical and experimental investigations of the Diffie-Hellman scheme security with respect to a directed search were conducted. In the case of a prime number of the form $p = 2q+1$, q is prime, the formulae for the expectation of the relative number of keys and γ-quantiles of the distribution of the number of keys necessary for the directed search are obtained. It is shown that in the case of directed search, provided that $q \to \infty$, the attacker needs to look through $\frac{3}{8}$ of the total number of keys, and the γ-quantile of the distribution of the relative number of keys tends to $\frac{2}{3}\gamma$. Is found that for the schemes for which the ununiformity of a shared secret key distribution significantly accelerate the directed search, the numbers $p-1$ are smooth. Experimental studies using developed software programs confirm the theoretical results.

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Institute of Physics and Technology of NTUU "KPI"

E-mail address: `mmzi@pti.kpi.ua`

PROVABLE SECURITY AGAINST DIFFERENTIAL ANALYSIS FOR SOME CLASSES OF NON-MARKOV SPN STRUCTURES

Sergey Yakovlev

In 2000-2001 Hong, Kang et al. gave theoretical upper bounds of differential probabilities for AES-like ciphers [1,2], but their results are applied only to Markov SPN. In this work we extend results of Hong, Kang et al. to the wide class of non-Markov SPN ciphers.

Let V_t be a linear space of t-bit vectors, and consider two sets of operations $\{\circ_i\}$, $\{\bullet_i\}$, $i = \overline{1,n}$, so $\langle V_t, \circ_i \rangle$ and $\langle V_t, \bullet_i \rangle$ are abellian groups with zero vector as identity element. Introduced operations induce $\circ$, $\bullet$ operations on V_t^m: $x \circ y = (x_1 \circ_1 y_1, \ldots, x_m \circ_m y_m)$, $x \bullet y = (x_1 \bullet_1 y_1, \ldots, x_m \bullet_m y_m)$, where $x, y \in V_t^m$, $x_i, y_i \in V_t$.

Consider one round of SP-network as $F(X, K) = L(S(X \bullet K))$, where $X \in V_t^m$ is an input data, $K \in V_t^m$ is a round key, $S = (s_1, ..., s_m)$ is S-box layer and L is a linear (with respect to $\circ$) mapping. The SPN cipher E_K is a composition of r round functions with independent and uniformly distributed round keys.

Let D_i be the maximum of differential probability of i-th S-box if the input and output differences are made with $\bullet_i$ and $\circ_i$ operations correspondingly, and let $D = \max_i D_i$. Let $D(E)$ be the maximum of differential probability of E_K over $\circ$ operation on input and output.

Our contribution is the following. For the cipher E_K with two rounds we estimate $D(E) \leq D^{B(L)-1}$, where $B(L)$ is the branch number of L over V_t. Moreover, if L is AES-like linear mapping, then for E_K with four rounds we have: $D(E) \leq D^{(B(L)-1)^2}$.

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National Technical University of Ukraine "Kiev Polytechnical Institute", Institute of Physics and Technology
E-mail address: `yasv@rl.kiev.ua`

LIMIT THEOREMS FOR STOCHASTIC PROCESSES AND RANDOM FIELDS

ASYMPTOTIC BEHAVIOR OF EXTREME VALUES IN SOME BANACH LATTICES

K. Akbash[1], I. Matsak[2]

Let (X_i), $i \geq 1$, be a sequence of independent copies of random element X assuming values in separable Banach lattice B with a module $|\cdot|$, $Z_n = \max\limits_{1\leq i\leq n} X_i$, $Z_n^* = \max\limits_{1\leq i\leq n} |X_i|$. Let $\varphi(x) = x^p L(x)$ is monotonically-increasing continuous function, $L(x)$ is slowly varying as $x \to \infty$.

In the report the conditions will be given for a space B and random element X, for which the following convergence almost surely will have a place:

$$\lim_{n\geq 1} \frac{\|Z_n^*\|}{\varphi^{-1}(n)} = 0 \text{ a.s.} \tag{1}$$

$$\lim_{n\to\infty} \left\| \frac{Z_n}{a_n} - \mathfrak{S}X \right\| = 0 \quad \text{a.s.} \tag{2}$$

$$\lim_{n\to\infty} \|Z_n - a_n \mathfrak{S}X\| = 0 \quad \text{a.s.} \tag{3}$$

Besides a convergence in the norm, an ordinal convergence in the ratio (1)-(3) will be also defined.

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National Taras Shevchenko University of Kyiv, Glushkova Ave. 2, corp. 6, Kiev, 03127
E-mail address: [1]`k_m_s_kirovograd@mail.ru`, [2]`ivanmatsak@gmail.com`

REDUCED BRANCHING PROCESSES WITH ARBITRARY NUMBER OF TYPES OF PARTICLES

I.B. Bazylevych, S.V. Mahun

Let $\mu_x(t)$ be a discrete time branching process with arbitrary number of types of particles. The set of all types of particles we denote by X, $\Im$ is a σ-algebra of subsets of X. $\mu_x(t)(t = 0,\ 1,\ 2, \ldots)$ is the number of particles at moment t, if at moment $t = 0$ we have one particle of type $x \in X$, $\mu_x(t, E)$ is a number of particles at moment t in the subset $E \in \Im$. $F(x, t, s(\cdot)) = M \exp\{\int_x \log s(z)\mu_x(t, dz)\}$ is a generation functional of the process $\mu_x(t)$. Probability of the degeneration to the moment t equals $q(x, t)$ and probability of degeneration of the process equals $q(x)$. Obviously, $q(x, t) = F(x, t, 0)$ [2]. $\mu_x(t; t + \tau)$, $t \geqslant 0$, $\tau > 0$ is the reduced process with process $\mu_x(t)$ [1]. A random measure $\mu_x(t; t + \tau, E)$ is the number of particles at the moment t with types in the subset $E \in \Im$. Let

$$F(x, t, \tau, s(\cdot)) = M[\exp(\int_X \log s(z)\mu_x(t; t + \tau, dz))],\ |s| \leqslant 1,$$

where $s(\cdot)$ is measurable function. $A(x, t, E)$ is first and factorial moments of process $\mu_x(t)$, $A(x, t, \tau, E)$ is first factorial moment of process $\mu_x(t, t + \tau)$. Denote

$$\mu_x(1) = \mu_x, A(x, 1, E) = A(x, E), A(x, X) = A(x), \underline{A} = \inf_{x\in X} A(x).$$

Theorem 1. *Let $\underline{A} > 1$, then for all $t = 0,\ 1,\ 2, \ldots$, $\tau = 0,\ 1,\ 2, \ldots$*

$$\lim_{\tau\to\infty} M[\exp(\int_X \log s(z)\mu_x(t, t + \tau, dz))|\mu_x(t + \tau) > 0] =$$

$$= \frac{F(x, t, s(\cdot)(1 - q(x)) + q(x)) - q(x)}{1 - q(x)}.$$

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Ivan Franko National University of L'viv, Universytetska, 1
E-mail address: I_Bazylevych@yahoo.com

NEW STATISTICAL METHODS IN SNP DATA ANALYSIS

A.V. Bulinski

The challenging problem in modern Medicine is to estimate the risks of complex diseases for individuals taking into account their SNP (single nucleotide polymorphisms) data and also the environmental factors. On this way the main achievements are based on the deep results established in bioinformatics, artificial intelligence, econometrics and mathematical statistics. In development of recent paper [1] (see also a number of references therein) the main attention will be paid to the new version of the MDR-method, various modifications of logic regression and machine learning methods. New theorems are proved to justify different methods of data analysis in the framework of specified stochastic models. In particular, we discuss the identification of the most significant combinations of genetic and nongenetic factors which could increase the risk of a disease. We apply the techniques of random trees and stochastic optimization as well as random fields theory. The applications to analysis of cardio-vascular diseases are provided. For this aim the supercomputer "Chebyshev" (MSU) was employed.

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Faculty of Mathematics and Mechanics, Lomonosov Moscow State University, Leninskie gory 1, Moscow, 119991, Russia
E-mail address: bulinski@mech.math.msu.su

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ON ERGODIC PROPERTIES OF NONLINEAR MARKOV CHAINS

O.A. Butkovsky

We investigate ergodic properties of nonlinear Markov chains defined on an arbitrary state space. These processes (in continuous time) were introduced by H.P. McKean in [2]. A detailed treatment of the subject is given in a recent monograph of V.N. Kolokoltsov [2]. Motivated by Muzychka and Vaninsky [3], who considered a nonlinear random walk and showed that nonlinear Markov processes may have unusual ergodic properties (e.g. continuum of stationary measures), we develop sufficient conditions for uniform ergodicity and existence and uniqueness of stationary measure. The conditions are optimal in a certain sense.

Let $(E,\mathcal{E})$ be a measurable space and denote by $\mathcal{P}(E)$ the class of all probability measures on this space. Let $X=(X_n^\mu)_{n\in\mathbb{Z}_+}$ be a nonlinear Markov process on $(E,\mathcal{E})$ with initial distribution $\mathrm{Law}(X_0^\mu)=\mu$, $\mu\in\mathcal{P}(E)$ and transitional probabilities $\mathrm{P}\big(X_{n+1}^\mu\in B|X_n^\mu=x\big)=P(X,B,\mu_n)$, where $x\in E$, $B\in\mathcal{E}$, $n\in\mathbb{Z}_+$ and $\mu_n:=\mathrm{Law}(X_n^\mu)$. By d_{TV} denote the total variation distance between two measures.

Theorem 1. *Suppose there exist $\varepsilon>0$ such that the process X satisfies Dobrushin condition, i.e.*

$$\sup_{\mu,\nu\in\mathcal{P}(E)}\sup_{x,y\in E} d_{TV}\big(P(x,\cdot,\mu),P(y,\cdot,\nu)\big)\leqslant 2(1-\varepsilon). \tag{1}$$

Moreover, assume that for some $\lambda\in[0,\varepsilon)$ and for all $x\in E$, $\mu,\nu\in\mathcal{P}(E)$ one has

$$d_{TV}\big(P(x,\cdot,\mu),P(x,\cdot,\nu)\big)\leqslant \lambda\, d_{TV}(\mu,\nu). \tag{2}$$

Then the process X is strongly ergodic, it has a unique stationary distribution, and for all $\mu,\nu\in\mathcal{P}(E)$ we have the following estimate of convergence rate

$$d_{TV}(X_n^\mu,X_n^\nu)\leqslant 2(1-(\varepsilon-\lambda))^n.$$

Remark 2. The condition $\lambda<\varepsilon$ is optimal. Namely, for any $0<\varepsilon<\lambda\leqslant 1$ there exist processes X, Y, Z that satisfy conditions (1) and (2), and measures $\mu,\nu\in\mathcal{P}(E)$ such that the process X has more than one stationary measure, the process Y has no stationary measures, $d_{TV}(Z_n^\mu,Z_n^\nu)\not\to 0$, as $n\to\infty$.

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Dept. of Mathematics and Mechanics, Moscow State University
E-mail address: oleg.butkovskiy@gmail.com

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LIMIT THEOREM FOR RANDOM BRANCHING PROCESS

G.B. Dzhufer

Consider random branching process with an arbitrary number of types T of particles, which transformation depends on their age and discrete time. In space T it is given σ-algebra of sets U.

Distribution function of life of one particle of type t is denoted by

$$p_k(t) = P_t\{r = k\},$$

where r is a lifetime of particle, P_t is the conditional probability that in initial moment of time there was one particle of type t, $\varphi(u)$ is a positive bounded function u-measurable function such that $0 < \varphi(u) \le 1$, $\xi_n(u)$ – number of particles in time n.,whose types belong to set $u \in U$. It should be noted that $\xi_n(u)$ – random measure. It is proved that $\lim\limits_{n\to\infty} nP_t \frac{1}{\Pi} \int\limits_T \varphi(u)\xi_n(u) \ge x$ has exponential distribution for $x > 0$ [1].

It is shown that by taking certain conditions already mentioned limit coincides with some infinitely divisible distribution.

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Ivan Franko National University of L'viv, Department of Theoretical and Applied Statistics, Lviv, Universytetska, 1, 79000
E-mail address: tumkiv_gala@mail.ru

NOTE ON THE SEVASTYANOV THEOREM

P. Endovytskii

Consider the following classical urn problem. Let n balls are successively thrown, independently and uniformly into m urns. Let $\mu_0 = \mu_0(n, m)$ be the number of empty urns, i.e. urns without balls. Well known [1] the next limit law for random variable μ_0.

Theorem 1. *Let $n = m\,log\,m + a\,m + o(m)$, $m \to \infty$, where $a \in R$ is some real constant. Then $\mu_0(n, m)$ weakly converges to Poisson distribution with parameter $exp\,(-a)$, when $n, m \to \infty$.*

In the case independent, but not necessary uniform allocation, theorem 1 is generalized in the following theorem [1].

Theorem 2. *Let n balls independently fall into m urns with probabilities $p_1, \dots, p_m$, $\sum_{i=1}^m = 1$. If $\sum_{i=1}^m (1-p_i)^n \to \lambda > 0$, $n, m \to \infty$, and $n\,min\,p_i \to \infty$, then $\mu_0(n, m)$ weakly converges to Poisson distribution with parameter λ, when $n, m \to \infty$.*

Theorem 2 doesn't give explicit expression for number of balls n as in theorem 1.

Consider one case in the theorem 2, when probabilities $p_1, \dots, p_m$ are defined via positive continuous function $p\,(x) \in C^{(2)}([0, 1])$, such, that $\int\limits_0^1 p\,(x)\,dx = 1$, and $p_i = \int_{\frac{i-1}{m}}^{\frac{i}{m}} p\,(x)\,dx$, $1 \leqslant i \leqslant m$. When $p\,(x) = 1$, $x \in [0, 1]$, we get uniform distribution. Next theorem is true.

Theorem 3. *Let $p\,(x)$ reachs its minimal value in the unique internal point $x_0 \in (0, 1)$ and $p''(x_0) > 0$. If $n = \frac{1}{p(x_0)}\,m\,log\,m - \frac{1}{2p(x_0)}\,m\,log\,log\,m + a\,m + o(m)$, $m \to \infty$, where $a \in R$ is some real constant. Then $\mu_0(n, m)$ weakly converges to Poisson distribution with parameter $exp\left(-a\,p\,(x_0)\sqrt{\frac{2\pi p\,(x_0)}{p''(x_0)}}\right)$, when $n, m \to \infty$.*

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Institute of Physics and Technology of NTUU "KPI", Kyiv, Ukraine
E-mail address: PavelEndow@ukr.net

ON A GENERAL APPROACH TO THE STRONG LAWS OF LARGE NUMBERS

István Fazekas

A general method to obtain strong laws of large numbers is studied. The method is based on an abstract Hájek–Rényi type maximal inequality.

We say that the random variables $X_1, \dots, X_n$ satisfy the Kolmogorov type maximal inequality for moments, if for each m with $1 \le m \le n$

$$\mathbb{E}\Big[\max_{1\le l\le m} |S_l|\Big]^r \le K \sum\nolimits_{l=1}^{m} \alpha_l \tag{1}$$

where $S_l = \sum_{i=1}^{l} X_i$, $\alpha_1, \dots, \alpha_n$ are non-negative numbers, $r > 0$, and $K > 0$.

We say that the random variables $X_1, \dots, X_n$ satisfy the Hájek-Rényi type maximal inequality for moments, if

$$\mathbb{E}\left[\max_{1\le l\le n} \left|\frac{S_l}{\beta_l}\right|\right]^r \le C \sum_{l=1}^{n} \frac{\alpha_l}{\beta_l^r} \tag{2}$$

where $\beta_1, \dots, \beta_n$ is a non-decreasing sequence of positive numbers, $\alpha_1, \dots, \alpha_n$ are non-negative numbers, $r > 0$, and $C > 0$.

In Fazekas and Klesov [1] it was shown that a Hájek-Rényi type maximal inequality for moments is always a consequence of an appropriate Kolmogorov type maximal inequality. Moreover, the Hájek-Rényi type maximal inequality automatically implies the strong law of large numbers. The most important is that no restriction is assumed on the dependence structure of the random variables.

In the paper we consider certain versions of the Hájek-Rényi inequality. Some applications for dependent random variables are given. The rate of convergence in the law of large numbers is also considered.

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Faculty of Informatics, University of Debrecen, P.O. Box 12, 4010 Debrecen, Hungary
E-mail address: fazekasi@inf.unideb.hu

PENULTIMATE APPROXIMATIONS IN STATISTICS OF EXTREMES AND RELIABILITY OF LARGE COHERENT SYSTEMS

M. Ivette Gomes, Paula Reis, Sandra Dias

In reliability theory any coherent system can be represented as either a series-parallel or a parallel-series system, and its lifetime can thus be written as the minimum of maxima or the maximum of minima. For large-scale coherent systems it is sensible to assume that the number of system components goes to infinity. Then, the possible non-degenerate extreme value laws either for maxima or for minima, established by Gnedenko (1943), are eligible candidates for the system reliability or at least for the finding of adequate lower and upper bounds for such a reliability. Reis and Canto e Castro (2009) have dealt with homogeneous series-parallel (or parallel-series) systems, identifying possible limit laws for the system reliability. However, it is well-known that in most situations such non-degenerate limit laws are better approximated by an adequate penultimate distribution. Dealing with regular and homogeneous parallel-series and series-parallel systems, and based on the results in Gomes and de Haan (1999), we try to assess the gain in accuracy when a penultimate approximation is used instead of a ultimate limiting approximation.

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CEAUL and DEIO, FCUL, Universidade de Lisboa
E-mail address: ivette.gomes@fc.ul.pt

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ABOUT THE BEHAVIOR OF CHARACTERISTIC FUNCTIONS OF PROBABILITY LAWS

O.M. Kinash, M.I. Parolya, M.M. Sheremeta

Let φ – analytic in $\mathbb{D}_R = \{z : |z| < R\}$, $0 < R \leq +\infty$, characteristic functions of probability law F, $M(r, \varphi) = \max\{|\varphi(z)| : |z| = r < R\}$ and $W_F(x) = 1 - F(x) + F(-x)$. In terms of generalized orders there are established the relations between the growth of$M(r, \varphi)$ and decrease of $W_F(x)$.

For entire characteristic functions is proved, for example, that if $\ln x_k \geq$ $\geq \lambda \ln\left(\frac{1}{x_k} \ln \frac{1}{W_F(x_k)}\right)$ for some increasing sequence (x_k) such that $x_{k+1} = O(x_k)$, $k \to \infty$, then $\ln \frac{\ln M(r,\varphi)}{r} \geq (1 + o(1))\lambda \ln r$ as $r \to +\infty$.

Lviv National Ivan Franko University, Department of Theoretical and Applied Statistics, Department of Theory of Functions and Probability Theory, 79000, Lviv, Universytetska,1

E-mail address: okinasch@yahoo.com, marta0691@rambler.ru, m_m_sheremeta@list.ru

B.V.GNEDENKO: CLASSIC OF LIMIT THEOREMS IN THE THEORY OF PROBABILITIES

V.S. Koroliuk

I. Limit distributions for sums of increment random variables.
II. Limit theorems for semimartingales.
III. Convergence of random evolutions in the scheme of Poisson approximation.

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01601, IM NASU, Tereschekivska str. 3, Kyiv, Ukraine

E-mail address: vskorol@yahoo.com

STRONG MARKOV APPROXIMATION OF THE SOLUTION OF LÉVY DRIVEN SDE BY SERIES SCHEME OF MARKOV CHAINS

T.I. Kosenkova

In this paper we present a generalisation of the scheme considered in [1] where strong Markov approximation was proved for a Lévy process without the diffusion component by the step processes assosiated to the triangullar array under the assumptions of Gnedenko's theorem (see [2]). The main result of this paper is as follows. Let $\{X_k^n,\ k \leq n\}, \quad n \geq 1$ be a sequence of Markov chains in $\mathbb{R}^d$. We consider the following process $X_n(t) = \sum_{k=1}^{n-1} X_k^n \cdot \mathbf{1}_{t \in [t_{kn}; t_{(k+1)n})}$. Let the family $\{\Pi(x, dy)\}, x \in \mathbb{R}^d$ be defined by a measure Π_0 and a bijective function $c : \mathbb{R}^d \times \Theta \to \mathbb{R}^d$ as follows

$$\Pi(x, A) = \Pi_0\{\theta : c(x, \theta) \in A\}.$$

Denote the process $X(t)$ by the solution of the following SDE

$$\begin{aligned} X(t) = X(0) + \int_0^t a(X(s-))ds + \int_0^t \int_\Theta c_1\left(X(s-), u\right) \tilde{\nu}(du, ds) + \\ + \int_0^t \int_\Theta c_2\left(X(s-), u\right) \nu(du, ds), \quad \text{where} \end{aligned} \tag{1}$$

$$c_1(x, u) = c(x, u)\mathbf{1}_{\|c(x,u)\| \leq 1}, \quad c_2(x, u) = c(x, u)\mathbf{1}_{\|c(x,u)\| > 1},$$

and ν is the Poisson point measure with the intensity measure Π_0.

Theorem 1. *Under the conditions analogous to those of Gnedenko's theorem, and assumig Lipschitz condition and the linear growth rate condition for the coefficiens of the equation (1), we obtain that the sequence X_n provides strong Markov approximation of the process X.*

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: tanya.kosenkova@gmail.com

LIMIT THEOREMS FOR MULTI-CHANNEL NETWORKS IN HEAVY TRAFFIC

A.V. Livinska

We consider the queuing network that consists of "r" service nodes. An input Poisson flow of calls with the leading function $\Lambda_i(t)$ arrives at i-th node of the network. Each of the "r" nodes acts as a multi-channel stochastic system. If a call arrives in such a system then its service immediately begins. Service time in i-th node is exponential distributed with the parameter $\mu_i, i = 1, 2, \ldots, r$. After service in i-th node the call with probability p_{ij} arrives to service at j-th node of the network and with the probability $p_{ir+1} = 1 - \sum_{j=1}^{r} p_{ij}$ leaves the network, $P = \|p_{ij}\|_1^r$ is the switching matrix of the network. Additional node numbered "$r + 1$" is interpreted as "output" from the network. At the initial time network is empty.

Let $Q_i(t), i = 1, 2, \ldots, r$ be the number of calls in i-th node of the network at the t moment of time and r-dimensional process $Q'(t) = (Q_1(t), \ldots, Q_r(t))$ will be called as the service process in the network.

It is shown that in heavy traffic the process $Q(t)$ is approximated by the Gaussian process $\xi^{(1)}(t) + \xi^{(2)}(t)$, $t \geqslant 0$, where $\xi^{(i)\prime}(t) = (\xi_1^{(i)}(t), \ldots, \xi_r^{(i)}(t))$, $i = 1, 2$, – two independent Gaussian processes, moreover $\xi^{(1)}(t)$ is associated with fluctuations of the input flows, and $\xi^{(2)}(t)$ – with fluctuations of service times. The test from [1] allows to check that the Gaussian process $\xi^{(1)}(t) + \xi^{(2)}(t)$ is diffusion. Justification is given as a functional limit theorem. Thus, the presented results generalize the material contained in Section 4.2 of the monograph [2], in case of Poisson input flows with variable rates.

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Kyiv National Taras Shevchenko University, Volodymyrska str. 64, Kyiv

E-mail address: livinskaav@gmail.com

ASYMPTOTICS OF BETA-COALESCENTS

Alexander Marynych

Exchangeable coalescent with multiple collisions, also known as Λ-coalescent is a Markovian process on the space of partitions of set $\{1, 2, \ldots, n\}$ into blocks. A block-counting process associated with the Λ-coalescent is a Markov chain $\Pi_n = (\Pi_n(t))_{t\geq 0}$ with right-continuous paths, it starts in state n when there are n singleton blocks in the partition and terminates in state 1 when a sole block remains. The blocks merge according to the rule: for each $t \geq 0$ when the number of blocks is $\Pi_n(t) = m > 1$, each k tuple of them is merging in one block at probability rate

$$\lambda_{m,\,k} = \int_0^1 x^k (1-x)^{m-k} x^{-2} \Lambda(\mathrm{d}x), \qquad 2 \leq k \leq m, \tag{1}$$

where Λ is a given finite measure on the unit interval.

The subclass of *beta-coalescents* are the processes driven by some beta measure on $[0, 1]$, with density

$$\Lambda(\mathrm{d}x)/\mathrm{d}x = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, \quad a, b > 0, \tag{2}$$

where $B(\cdot, \cdot)$ denotes Euler's beta function.

In this talk we focus on the class of beta-coalescents with parameter $0 < a \leq 1$. Specifically, we are interested in the number of collisions X_n which is equal to the total number of particles born by collisions and the total branch length of the coalescent tree L_n which is the cumulative lifetime of all particles from the start of the process to its termination.

We will show how to combine the method of probability metrics and results from the renewal theory to prove the weak convergence of X_n and L_n as $n \to \infty$. Our results complement previously known facts concerning the weak convergence of mentioned functionals in case $a > 1$, obtained via completely different techniques.

The talk is based on the joint work with A. Iksanov and A. Gnedin.

Eindhoven University of Technology, Department of Mathematics and Computer Science, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

E-mail address: O.Marynych@tue.nl

INVARIANCE PROPERTIES OF RANDOM VECTORS AND STOCHASTIC PROCESSES BASED ON THE ZONOID CONCEPT

I. Molchanov, M. Schmutz, K. Stucki

With each random vector X it is possible to associate a convex body called the zonoid of X and which is defined as the expectation of the random segment with end-points being the origin and X. It is possible that two different random vectors share the same zonoid, which leads to the idea of zonoid equivalent random vectors. The idea of zonoid equivalence stems also from financial applications, see [1].

The talk addresses the properties of zonoid equivalent vectors and also related facts for stochastic processes through their finite-dimensional distributions. I particular, generalisations of the stationarity and exchangeability concepts are explained, see [2].

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Institute of Mathematical Statistics and Actuarial Science, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland
E-mail address: ilya.molchanov@stat.unibe.ch, michael.schmutz@stat.unibe.ch, kaspar.stucki@stat.unibe.ch

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ON THE RATE OF CONVERGENCE IN THE UNIFORM LAW OF LARGE NUMBERS

V.I. Norkin

A uniform law of large numbers states uniform convergence of the sample average approximation functions $f^N(x,\xi^N) = \frac{1}{N}\sum_{i=1}^N f(x,\xi_i)$ to the expectation function $\mathbf{E}f(x,\xi)$ on a set $X \subseteq R^n$ for iid random arguments $\{\xi_i : \Omega \to \Xi, i = 1,2,...\}$ distributed as a random variable $\xi : \Omega \to \Xi$ and defined on a common probability space $(\Omega, \Sigma, \mathbf{P})$; $\xi^N = (\xi_1, ..., \xi_N)$.

In the report we develop Talagrand's [1] type inequalities for maximal deviations of f^N from $\mathbf{E}f$, $\Delta_N = \sup_{x\in X}\left|f^N(x,\xi^N) - \mathbf{E}f(x,\xi)\right|$, under varies assumptions on the random function $f(\cdot,\xi)$. We consider cases of discrete sets X or Ξ, Hölder continuous function $f(x,\xi)$ in x or ξ [2]. As an example we formulate the following result.

Theorem 1. *Assume that* **(i)** f *is Lipschitzian in* $x \in X$ *uniformly in* $\xi \in \Xi$, *i.e.* $|f(x_1,\xi) - f(x_2,\xi)| \le L\|x_1 - x_2\|$ *for all* $x_1, x_2 \in X$, $\xi \in \Xi$; **(ii)** f *is uniformly bounded,* $\sup_{x\in X,\,\xi\in\Xi}|f(x,\xi)| \le M < \infty$; **(iii)** $X \subset R^n$ *is bounded with diameter* $D_X\sqrt{n}$.

Then for any $t \ge 0$ *and* $N = 1,2,...$ *holds*

$$\mathbf{E}\Delta_N \le LD_X\sqrt{n}(1+\sqrt{n\ln N})/N^{1/2},$$

$$\mathbf{P}\left\{\Delta_N \ge t + \frac{LD_X\sqrt{n}(1+\sqrt{n\ln N})}{N^{1/2}}\right\} \le 2\exp\left\{-\frac{t^2}{2M^2}\right\}.$$

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Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, Glushkov avenue 40, Kiev, Ukraine
E-mail address: norkin@i.com.ua

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ON THE LIMIT BEHAVIOR OF A SYMMETRIC RANDOM WALK WITH PERTURBATIONS AT THE ORIGIN

A.Yu. Pilipenko[1], Yu.E. Prykhodko[2]

Let $(X_1(k),\ k\in\mathbb{Z}_+)$ be a homogeneous Markov chain on $\mathbb{Z}$ with transition probabilities $p_{i,j}$ such that for some fixed m

$$p_{i,i+1} = p_{i,i-1} = 1/2 \text{ for } |i| > m \quad \text{and} \quad \sum\nolimits_j |j|\,p_{i,j} < \infty \text{ for } |i| \leqslant m.$$

For each natural $n \geqslant 2$ we consider a process $X_n(t) = \frac{1}{\sqrt{n}}X_1(nt),\ t\geqslant 0$, where $(X_1(t),\ t\geqslant 0)$ is the linear interpolation of $(X_1(k),\ k\in\mathbb{Z}_+)$.

Consider a stopping time $\tau = \inf\{k\colon |X_1(k)| > m\}$. By $\xi^{(\pm)}$ denote the distribution of $\big(X_1(\tau) - m\,\mathrm{sign}\,X_1(\tau)\big)$ given that $X_1(0) = \pm m$.

Theorem 1. *The sequence* $\{X_n\}$ *converges weakly in* $\mathbb{C}([0,1])$ *to a continuous process* X_∞. *In particular, if the chain* $(X_1(k),\ k\in\mathbb{Z}_+)$ *is recurrent, then* X_∞ *is a skew Brownian motion* W_γ [1] *with the parameter*

$$\gamma = \frac{\mathrm{E}\xi^{(+)}\,\mathrm{P}(\xi^{(-)}>0) + \mathrm{E}\xi^{(-)}\,\mathrm{P}(\xi^{(+)}<0)}{\mathrm{E}|\xi^{(+)}|\mathrm{P}(\xi^{(-)}>0) + \mathrm{E}|\xi^{(-)}|\mathrm{P}(\xi^{(+)}<0)}.$$

For $m = 0$ we obtain the result stated in [1], see also [2], [3].

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[1] Institute of Mathematics of Ukrainian National Academy of Science, 3 Tereschenkivska str., Kiev, 01601, Ukraine
E-mail address: apilip@imath.kiev.ua

[2] National Technical University of Ukraine "KPI", 37 Peremohy ave., Kiev, Ukraine
E-mail address: npuxodbko@gmail.com

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ASYMPTOTIC PROPERTIES OF FUNCTIONAL PARAMETER ESTIMATORS OF TIME AND SPATIAL HILBERT-VALUED SERIES MODELS

M.D. Ruiz-Medina

The autoregressive Hilbertian time series (ARH(p) series) framework has been introduced in Bosq (2000) and Bosq and Blanke (2007). Spatiotemporal data models displaying weak-dependence are studied under this perspective in Salmerón and Ruiz-Medina-Medina (2009). Moment-based parameter estimation has been addressed in Bosq (2000) and Bosq and Blanke (2007). Projection maximum-likelihood parameter estimators are derived, in the Gaussian case, in Ruiz-Medina and Salmerón (2010). In this paper, the asymptotic Gaussian distribution of maximum likelihood estimators is obtained, in the case where the distribution of the functional innovation process belongs to the exponential family. In Ruiz-Medina (2011), spatial autoregressive Hilbertian processes (SARH processes) are introduced under stationarity and weak dependence of the Hilbert-valued innovation process. Some limit results are derived in the separable case. In this paper, asymptotic properties of moment-based functional parameter estimators are investigated, extending the methodology applied in Bosq (2000) for the ARH(p) case.

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Department of Statistics and Operations Research, Faculty of Sciences, University of Granada, Campus Fuente Nueva s/n, 18071 Granada, Spain.
E-mail address: mruiz@ugr.es

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SERIES OF REGRESSIVE SEQUENCES

M.K. Runovska

Consider the regressive sequence of random variables (η_k):

$$\eta_1 = \beta_1\theta_1, \quad \eta_k = \alpha_k\eta_{k-1} + \beta_k\theta_k, \quad k \geqslant 2, \tag{1}$$

where (α_k) and (β_k) are nonrandom real sequences, (θ_k) is a sequence of independent symmetric random variables such that $\mathbb{P}\{\theta_k = 0\} < 1, \ \ k \geq 1$. In [1] the criteria for the convergence almost surely (a.s.) of series $\sum_{k=1}^{\infty}\eta_k$ for the sequence (η_k) generated by a standard Gaussian sequence (θ_k) was found. The Theorem 1 gives a generalization of this result on the case of any regressive sequence (η_k) of kind (1). The proof of Theorem 1 is based on the theory of random series in Banach spaces (see [2]).

Theorem 1. *In order for the series $\sum_{k=1}^{\infty}\eta_k$ to converge a.s. it is necessary and sufficient that the following three conditions be satisfied*:

1) *the series $\sum_{l=1}^{\infty}\beta_k\Big(\prod_{j=k+1}^{k+l}\alpha_j\Big)$ is convergent for any $k \geq 1$;*

2) *the random series $\sum_{k=1}^{\infty}A(\infty,k)\theta_k$ is convergent a.s., where $A(\infty,k) = \beta_k + \sum_{l=1}^{\infty}\beta_k\Big(\prod_{j=k+1}^{k+l}\alpha_j\Big)$;*

3) *for all the sequences $(m_j) \in \mathfrak{R}^{\infty}$ one has $\Big|\sum_{k=m_j+1}^{m_{j+1}}A(m_{j+1},k)\theta_k\Big| \underset{j\to\infty}{\to} 0$, a.s., where*

$$A(n,k) = \begin{cases} 0, & n < k; \\ \beta_k, & n = k; \\ \beta_k + \sum_{l=1}^{n-k}\beta_k\Big(\prod_{j=k+1}^{k+l}\alpha_j\Big), & n > k. \end{cases}$$

Moreover, if conditions 1)—3) *hold then* $\sum_{k=1}^{\infty} \eta_k = \sum_{k=1}^{\infty} A(\infty, k)\theta_k, \quad a.s.$

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Department of mathematical analysis and probability theory, National Technical University of Ukraine ("KPI"), prosp. Peremogy, 37, Kiev, 03056, Ukraine

E-mail address: mari-run@ukr.net

STRATIFIED MONTE CARLO QUADRATURE FOR CONTINUOUS RANDOM FIELDS

O. Seleznjev, K. Abramowicz

We consider the problem of numerical approximation of integrals of random fields over a unit hypercube. We use a stratified Monte Carlo quadrature and measure the approximation performance by the mean squared error. We focus on random fields satisfying a local stationarity condition proposed for stochastic processes by [3] and extended for random fields in [2]. Approximation of random functions from this class is studied in, e.g., [2, 1] . The quadrature is defined by a finite number of stratified randomly chosen observations with the partition (or strata) generated by a rectangular grid (or design). We study the class of locally stationary random fields whose local behavior is like a fractional Brownian field in the mean square sense and find the asymptotic approximation accuracy for a sequence of designs for large number of the observations. For the Hölder class of random functions, we provide an upper bound for the approximation error. Additionally, for a certain class of isotropic random functions with an isolated singularity at the origin, we construct a sequence of designs eliminating the effect of the singularity point.

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Department of Mathematics and Mathematical Statistics, Ume University, SE-901 87, Ume, Sweden

E-mail address: oleg.seleznjev@math.umu.se, konrad.abramowicz@math.umu.se

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A FUNCTIONAL LIMIT THEOREM FOR THE BOUNDARY MEASURE OF GAUSSIAN EXCURSION SETS

A. Shashkin

Gaussian excursion sets have drawn much attention recently due to their applications in image analysis, tomography and astrophysics. It is natural to analyze the behavior of the geometric functionals of these random sets, such as volumes, surface areas etc. Starting from the celebrated Rice formula, many more involved results on moments computation and limit theorems for these functionals have been established. They are due to Cramér, Belyaev, Cuzick, Piterbarg, Kratz and many other scientists, see [1] and references therein. Considering any of these geometrical characteristics for all real levels simultaneously, one obtains a random process. In this abstract we provide a functional central limit theorem for the random process of level set measures obtained in this way.

Let $d > 2$ and $X = \{X(s), s \in \mathbb{R}^d\}$ be a centered, stationary, isotropic and pathwise C^2 Gaussian random field with covariance function R which is C^4. Suppose also that R and all its derivatives up to order two belong to $L^1(\mathbb{R}^d)$. Without loss of generality we require also that the variance of X as well as of all its first-order derivatives equals 1. Denote by $\mathcal{H}_k$ the k-dimensional Hausdorff measure of a bounded Borel subset of $\mathbb{R}^d$. For $u \in \mathbb{R}$ and $t > 0$, set $N_t(u) := \mathcal{H}_{d-1}(\{s \in [0,t]^d : \ X(s) = u\})$ and take $Z_t(u) := (Z_t(u) - \mathsf{E}Z_t(u))/t^{d/2}$, where $t > 0$ and $u \in \mathbb{R}$. Then one has

Theorem 1. *The random processes $Z_t(\cdot)$ converge in distribution in $C(\mathbb{R})$, as $t \to \infty$, to a centered Gaussian process with covariance function determined by that of X.*

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Department of Probability Theory, Faculty of Mathematics and Mechanics, Moscow State University, Leninskie Gory, GSP-1, Moscow, 119991, Russia

E-mail address: ashashkin@hotmail.com

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LIMIT THEOREMS FOR EXCURSION SETS OF STATIONARY RANDOM FIELDS

Evgeny Spodarev

In this talk, we give an overview of the recent asymptotic results on the geometry of excursion sets of stationary random fields. Namely, we present a number of limit theorems of central type for the volume of excursions of stationary (quasi–, positively or negatively) associated random fields with stochastically continuous realisations for a fixed excursion level. This class includes in particular Gaussian, Poisson shot noise, certain infinitely divisible, α–stable and max–stable random fields satisfying some extra dependence conditions. Functional limit theorems (with the excursion level being an argument of the limiting Gaussian process) are proved as well. For stationary isotropic C^1–smooth Gaussian random fields similar results are available also for the surface area of the excursion set. The case of the excursion level increasing to infinity and the asymptotics of the mean volume, surface area and the Euler characteristic of excursions of non–stationary smooth Gaussian random fields with a unique point of maximum variance are considered as well.

Ulm Universtiy, Institute of Stochastics, 89069 Ulm, Germany
E-mail address: evgeny.spodarev@uni-ulm.de

GNEDENKO-STONE LOCAL LIMIT THEOREMS FOR RANDOM WALKS CONDITIONED TO STAY POSITIVE

Vladimir Vatutin[1], Vitali Wachtel[2]

Let $S_0 = 0, \{S_n\}_{n\geq 1}$ be a random walk generated by a sequence of i.i.d. random variables $X_1, X_2, ...$ and let $\tau^- = \min\{k \geq 1 : S_k \leq 0\}$. Assuming that the distribution of X_1 belongs to the domain of attraction of an α-stable law, $\alpha \neq 1$, with $\mathbb{E}X_1 = 0$ (if exists), we study the asymptotic behavior of the probabilities $\mathbb{P}(\tau^- = n)$ and $\mathbb{P}(S_n = x|\tau^- > n)$ as $n \to \infty$. The last is the version of the Gnedenko-Stone local limit theorems for random walks conditioned to stay positive. It is shown that the conditional local limit theorems are essentially different from the nonconditional ones.

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[1] Department of Discrete Mathematics, Steklov Mathematical Institute, Gubkin street 8, 119991, Moscow, Russia
E-mail address: vatutin@mi.ras.ru
[2] Mathematical Institute, University of Munich, Theresienstrasse 39, D–80333 Munich, Germany
E-mail address: wachtel@mathematik.uni-muenchen.de

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CONVERGENCE OF MARTINGALES TO A CONTINUOUS LÉVY-TYPE PROCESS

A. Yurachkivsky

For a càdlàg function f we denote $\Delta f(t) = f(t) - f(t-)$. The symbol $\xrightarrow{\mathrm{C}}$ signifies the convergence in law of càdlàg random processes to a continuous process. By $\sharp$ we denote the linear operation in the space of (0,4)-tensors acting on tensors products of vectors as follows: $(a_1 \otimes a_2 \otimes a_3 \otimes a_4)^\sharp = a_1 \otimes a_3 \otimes a_2 \otimes a_4$. Some applications of the main result of [1] will be given. Here is one of them.

Theorem 1. *Let for each $T > 0$ the random process X_T be defined by*

$$X_T(t) = \frac{1}{\sqrt{T}} \int_0^{Tt} \mathrm{d}M(s) \otimes \vartheta(s),$$

where M is an $\mathbb{R}^d$-valued càdlàg locally square integrable martingale and ϑ is an $\mathbb{F}$-predictable $\mathbb{R}^d$-valued random process such that $\int_0^t \|\vartheta(s)\|^2 \mathrm{d}\,\mathrm{tr}\langle M\rangle(s) < \infty$ for all t. Suppose also that the following conditions are fulfilled:

$$\lim_{T\to\infty} T^{-1} \mathsf{E} \max_{s\leqslant T} |\Delta M(s)|^2 = 0, \quad \lim_{L\to\infty} \overline{\lim_{T\to\infty}}\, \mathsf{P}\{\mathrm{tr}\langle M\rangle(T) > TL\} = 0;$$

there exists a family, indexed by $N \in \mathbb{N}$, of differentiable functions $h_N : \mathbb{R}^d \to \mathbb{R}_+$ with absolutely continuous derivative such that, firstly, $h_N(x) = 0$ as $|x| \leqslant N$, secondly, $\varliminf\limits_{|x|\to\infty} |x|^{-2} h_N(x) > 0$ and, thirdly, for any positive ε

$$\lim_{N\to\infty} \varlimsup_{T\to\infty} \mathsf{P}\left\{\int_0^T h_N(\vartheta(s))\,\mathrm{d}\,\mathrm{tr}\langle M\rangle(s) > T\varepsilon\right\} = 0;$$

there exists a random tensor G such that $\frac{1}{T}\int_0^T \mathrm{d}\langle M\rangle(s) \otimes \vartheta(s)^{\otimes 2} \xrightarrow{\mathrm{d}} G$. Then $X_T \xrightarrow{\mathrm{C}} X$, where X is a continuous local martingale with quadratic characteristic $\langle X\rangle(t) = G^\sharp t$.

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National Taras Shevchenko University of Kyiv, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: yap@univ.kiev.ua

ALMOST SURE APPROXIMATION OF THE SUPERPOSITION OFTHE RANDOM PROCESSES

N.M. Zinchenko

This work is the development of the author's previous works [1,2]. Here we present some general results concerning sufficient conditions for almost sure (a.s.) approximation of the superposition of the cad-lag random processes $S(N(t))$ by Wiener or α-stable Lévy process, when cad-lag random processes $S(t)$ and $N(t)$ themselves admit a.s. approximation by Wiener or stable Lévy processes. Such results can serve as a source of numerous strong limit theorems for the random sums of the type

$$D(t) = S(N(t)) = \sum_{i=1}^{N(t)} X_i$$

under various assumptions on counting point process $N(t)$, dependence and moment conditions of summands $\{X_i, i \geq 1\}$. For instance, $\{X_i, i \geq 1\}$ may be i.i.d.r.v or martingales, weakly dependent or associated sequences with certain conditions on Cox-Grimmett coefficient; we can regard $N(t)$ as a renewal process, i.e. $N(t) = \inf\{x \geq 0 : Z(x) > t\}$, $\quad Z(x) = \sum_{i=1}^{[x]} z_i$, where $\{z_i, i \geq 1\}$ also can satisfy various moment and dependence conditions. As a consequence we present a number of results concerning the a.s. approximation of the Kesten-Spitzer random walk in random scenery, storage processes, risk processes in the classical and renewal Sparre Andersen risk models with small and large claims, risk processes with stochastic premiums and use such results for investigation the growth rate and fluctuations of the mentioned processes.

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Nizhyn State Mukola Gogol University, Kropyv'yanskogo, 2, Nizhyn, Ukraine
E-mail address: znm@univ.kiev.ua

MARKOV AND SEMI – MARKOV PROCESSES

A NEW VIEW ON MIXING AND ITS APPLICATIONS TO PDES, SWITCHING SDES AND QUEUEING

S.V. Anulova[1], A.Yu. Veretennikov[2]

New results on the *convergence rate of a Markov chain to the stationary distribution* are obtained via a *modified Vaserstein's coupling* construction [2]. *Applications to PDEs* concern Poisson equation $Lu - cu = -f$ in R^d for an elliptic differential operator of the second order L. Here a potential c may be non-negative, or with non-trivial positive and negative parts, under a stability of the Markov diffusion with a generator L, or under large deviation conditions (cf. [7]). In *switching degenerate diffusions* (cf. [1]), a new exponential stability is established. In *queueing,* new *rates of convergence in Erlang–Sevastyanov's problem* (cf. [3, 3, 6]) are established. On existence of a stationary regime in the latter problem see [3, 6]; on exponential convergence cf. [4].

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[1] Inst. Control Sci., Russian Acad. Sci., 65 Profsoyuznaya, Moscow, Russia
E-mail address: anulovas@ipu.ru
[2] School of Mathematics, University of Leeds, LS2 9JT, UK & IITP, Moscow, Russia
E-mail address: a.veretennikov@leeds.ac.uk

POWER GENERALIZED WEIBULL MODEL: A BAYES STUDY USING MARKOV CHAIN MONTE CARLO SIMULATION

L. Azarang

In many clinical trails, life time distribution has a unimodal failure rate function. However, there are very few practical models to model this type of failure rate function. The Power generalized Weibull family introduced by Nikulin and Bagdonavicius (2002) can be useful in this case. The paper focuses on this model, Bayes estimators are discussed and MCMC simulation study is conducted to evaluate the performance of the estimation method. Usefulness and flexibility of the family is illustrated by analyzing a given data set(a real data set is considered for illustration).

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Department of Mathematics, Statistics, and Computer sciences, University of Tehran, Iran
E-mail address: azarangl@yahoo.com

THE SEQUENCE OF DEPENDENT EVENTS

Z.G. Boychuk, Ya.I. Yeleyko

Within the Bernoulli trial ([1]) problems, the independence of tests and the same probability p of the appearance of events A in each experiment may be caused by neglecting the time factor for a particular type of a problem. Such a neglect brings about erroneous predictions in social relationships. The following definition was presented for the first time in the paper [2]:
Definition 1: *The reaction of the event B to the event A that took place prior to the event B:* $R_A(B) = P_A(B) - P_{\overline{A}}(B)$.

Some properties of reaction R. For every events A, B:

1. $-1 \leq R_A(B) \leq 1$; a)$(R_A(B) = -1) \Leftrightarrow (B = \overline{A})$; b)$(R_A(B) = 1) \Leftrightarrow (B = A)$;
2. In general, $R_A(B) \neq R_B(A)$;
3. For independent events A, B: $R_A(B) = R_B(A) = 0$.

We can see from the above properties that the response reaction $R_A(B)$ may be the measure of the effect of one event A upon the following in time event B. We can now consider a series of n dependent events that are pairwise connected by arbitrary reactions $R_{A_i}(A_{i+1}), i = 1, 2, ..., n$. Unlike Markov chain, we have the chain of dependent events. Event A_{i+1} "has past experience" and the probability of its own appearance is changing according to the reaction $R_{A_i}(A_{i+1}), i = 1, 2, ..., n$.

The report describes the measure of the effect of the event on the subsequent events while researching the variable $SDA(m)$, that determines the number of Simple(Prime) Dividers Additional of integer m. Each integer m in the interval $(2^n, 2^{n+1}]$ contains no more than n simple additional divisors. For example: $SDA(prime) = 0$, $SDA(2^k) = k-1$, $SDA(140) = SDA(2*2*5*7) = 3$. Each integer within the interval $(2^n, 2^{n+1}]$ can be considered as a result of a sequence of n depending experiments, in each of with the natural number will receive simple multiplier additional, or may be not. $SDA(m)$ is not $\tau(m)$ function, because the latter determines the number of all divisors of m.

The report managed to find the reactions $R_{A_{i-1}}(A_i), i = 1, 2, ..., n$. There have been constructed distributions of the $SDA(m)$ variable on the discrete set $\{0, 1, ..., n\}$ for different n. These distributions are similar to the binomial distribution, but not as the last, because events in the sequences of this research are dependent.

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Ivan Franko National University of L'viv, Universytetska, 1, L'viv, Ukraine

E-mail address: Z_Boichuk@yahoo.com

ON THE APPLICATIONS OF THE STRONG LAW OF LARGE NUMBERS FOR A PIECEWISE FRACTIONAL LINEAR MAP WITH EXPLICIT INVARIANT MEASURE

Chrysoula Ganatsiou

A random system with complete connections is a particular case of infinite order chain. Another way for studying higher-order Markov chains is the construction of multiple Markov chains through collections of directed circuits and positive weights named as higher-order circuit chains. Following the concept of the theory of stochastic dependence with complete connections the present work arises as an attempt to investigate the problem of approximation of a nonzero irrational number y [0,1] by the n-th convergent of the corresponding expansion of y defined by a piecewise fractional linear map with explicit invariant measure. This will give us the possibility to obtain some important applications of the strong law of large numbers for the associated Markov chain of the corresponding random system with complete connections.

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University of Thessaly, School of Technological Sciences, Department of Civil Engineering, Section of Mathematics and Statistics, Pedion Areos, 38334, Volos, Greece

E-mail address: ganatsio@otenet.gr

SEMI-MARKOV APPROACH TO THE PROBLEM OF DELAYED REFLECTION OF DIFFUSION MARKOV PROCESSES

B.P. Harlamov

A homogeneous diffusion Markov process with an open interval as its range of values is considered. Extension of this process to the closure of this interval obtained by adding two its edge points is studied. This extension is made by law of continuous semi-Markov process when one appoints a distribution of the first exit time from any one-sided neighborhood of a boundary point. The admissible family of distributions for each boundary point is determined in terms of Laplace transforms to within an arbitrary function L from the set of all the non-decreasing functions of argument $\lambda \geqslant 0$ with completely monotone derivatives with respect to λ, and with property $L(0) = 0$. The process with the extended range of values is proved to be Markov if and only if the functions L_1 and L_2, corresponding to two boundary points are linear in their arguments. In particular, if both the functions are equal to zero identically, the process is Markov. The latter case corresponds to instantaneous reflection of the process from the boundaries of the interval. In other case, linear and non-linear, the reflection takes place with delay. One can find and measure such a delay with the help of determination of stationary probabilities for the process to stay at the boundaries of the interval. For the extended processes with finite derivatives $L_1'(0)$ and $L_2'(0)$ stationary one-dimensional distributions can be found by methods of renewal process theory. A special class of the extended processes is formed by those of them what have infinite derivative $L_1'(0)$ or $L_2'(0)$. In particular, a truncated Wiener process is such a process, where $L_1(\lambda) = L_2(\lambda) = \sqrt{\lambda}$. The results concerning with a diffusion Markov process reflecting with delay from boundaries of an interval can be generalized on an isolated point inside the interval (see "Zapiski nauchnykh seminarov POMI", v. 384, 2010, 292–310 (in Russian)).

In particular, a truncated Wiener process $\overline{w}$ is such a process, where $\overline{w}(t) = w(t)$ for $0 < w(t) < 1$; $\overline{w}(t) = 0$ for $w(t) \leqslant 1$; $\overline{w}(t) = 1$ for $w(t) \geqslant 1$. In this case it is not difficult to evaluate $L_1(\lambda) = L_2(\lambda) = \sqrt{\lambda}$ (with a positive factor). The results concerning with a diffusion Markov process reflecting with delay from boundaries of an interval can be generalized on an isolated point inside the interval (see "Zapiski nauchnykh seminarov POMI", v. 384, 2010, 292–310 (in Russian)). Applying to such a point the semi-Markov rule for transforming the process, we obtain a new process. In this case a family of distributions of the first exit points-times from all neighborhoods of this point is assigned according to some consistent method. Such a process is determined to within two arbitrary objects: 1) a function $L(\lambda)$ from the above mentioned class, which determines a delay of the process at this point; and 2) a non-negative number, which determines an asymmetry of the process while it goes through this point (the point behaves like a semiconductor).

Engelsa pr., 63/3, kv. 3, 194017, St.Petersburg, Russia
E-mail address: b.p.harlamov@gmail.com

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FLUCTUATION OF DIFFERENCE PROCEDURES STOCHASTIC OPTIMIZATION WITH IMPULSIVE PERTURBING

U.T. Himka, Ya.M. Chabanyuk

Consider the difference stochastic optimization procedure (SOP) with impulsive perturbation in the form

$$du^\varepsilon(t) = a(t)[\nabla_b C(u^\varepsilon(t), x(t/\varepsilon^4)) + \varepsilon d\eta^\varepsilon(t)], \tag{1}$$

of the regression function $C(u; \cdot) \in C^2(R)$ and pseudo-gradient $\nabla_b C(u, \cdot) = (C(u + b(t); \cdot) - C(u - b(t); \cdot)/2b(t)$, where depends on uniformly ergodic Markov process $x(t) > 0, t \geqslant 0$, in the dimensional phase space of states $(X, \mathbf{X})$, that is defined by the generator $Q\varphi(x) = q(x)\int_X P(x, dy)[\varphi(y) - \varphi(x)], \varphi \in B(X)$, where $B(X)$ the Banah space of real limited function withe norm $\|\varphi\| = \max\limits_{x \in X} |\varphi(x)|$, [1]. $P(x, B), x \in X, B \in \mathbf{X}$, - the stochastic kernel embedded Markov chain $x_n = x(\tau_n), n \geqslant 0$, withe by stationary distribution $\pi(B), B \in \mathbf{X}$ [1]. The impulsive process disturbances $\eta^\varepsilon(t), t \geqslant 0$, given by the relation $\eta^\varepsilon(t) = \int_0^t \eta^\varepsilon(ds; x(s/\varepsilon^4))$, where $\eta^\varepsilon(t, x), t \geqslant 0, x \in X$ - family of processes with independent increments, set the generator $\mathbf{\Gamma}^\varepsilon(x)\varphi(v) = \varepsilon^{-4}\int_R[\varphi(v + \varepsilon^2 w) - \varphi(v)]\Gamma(dw; x)$[2]. For the case $a(t) = a/t^\gamma, a > 0, 1/2 < \gamma < 1$, consider the fluctuation of the SOP (1) $v^\varepsilon(t) = \varepsilon^{-1}t^\delta[u^\varepsilon(t) - \varepsilon\mu^\varepsilon(t)]$, where $\mu^\varepsilon(t) = a\int_{t_0}^t \eta^\varepsilon(ds; x(s/\varepsilon^4))/s^\gamma$ – impulsive process perturbing [1].

Theorem 1. *When the terms of the balance condition* $\int_X \pi(dx)b_1(x) = 0, b_1(x) = \int_R v\Gamma(dv, x)$, $\int_X \pi(dx)C(0; x) = 0$, *there is a weak convergence*

$$v^\varepsilon(t), \mu^\varepsilon(t) \Rightarrow (\zeta(t), \sigma(t)W(t)), t > 0, \varepsilon \to 0,$$

in the interval $0 < t_0 < t < T$. *The limit process* $(\zeta(t), \sigma(t)W(t)), t > 0$, *generator set*

$$\mathbf{L}\varphi(v, w) = \left[v(act^{-\gamma} + \delta t^{\delta-1}) + act^{\delta-\gamma}w\right]\varphi_v'(v, w) + (a/2)B(t)\varphi_w''(v, w),$$

where $c = \int_X \pi(dx)\nabla C'(0, x)$; $B(t) = at^{-\gamma}B_1 + t^{\delta-\gamma}B_2$, $B_1 = 2\int_X \pi(dx)b_1(x)R_0b_1(x)$, $B_2 = \int_X \pi(dx)b_2(x)$, $b_2(x) = \int_R v^2\Gamma(dv; x)$.

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Lviv Polytechnic National Univercity, Applied Mathematics, 12 St. Bandera St., Lviv, Ukraine
E-mail address: yaroslav_chab@yahoo.com

ON JOINT LIMIT DISTRIBUTION LAW IN Q-PROCESSES

Azam A. Imomov, Erkin Tukhtayev

Let $\{Z_n,\ n \in \mathbf{N}_0\}$ $(\mathbf{N}_0 = \{0\} \cup \{\mathbf{N} = 1, 2, \ldots\})$ be a Galton-Watson branching Process (GWP) with probability generating function (GF) $F(s) = \mathbf{E}\left[s^{Z_1}\middle|\, Z_0 = 1\right]$.

Consider a family of random variables $\{W_n,\ n \in \mathbf{N}_0\}$ described by sequence of GF

$$\left\{W_n(s) := \mathbf{E}s^{W_n} = s\frac{F_n'(qs)}{\beta^n},\quad 0 \leqslant s < 1,\quad n \in \mathbf{N}_0\right\},$$

where $F_n(s) := \mathbf{E}s^{Z_n}$, $\beta = F'(q) < 1$, and quantity $q \in (0;1]$ is the extinction probability of GWP. The family $\{W_n,\ n \in \mathbf{N}_0\}$ is a Markov chain and represents the states sequence of Q-process at a moment n with transition probabilities

$$\mathbf{P}\left\{W_{n+k} = j \mid W_k = i\right\} = \frac{jq^{j-i}}{i\beta^n}\mathbf{P}\left\{Z_{n+k} = j \,|Z_k\, = i\right\},\ i,j,k \in \mathbf{N};$$

see [1, p.58] and [2].

We investigate a joint limit distribution law of W_n and

$$S_n := W_0 + W_1 + \ \ldots\ + W_{n-1}, \quad S_0 = 0.$$

The variable S_n means a total state in Q-process.

Theorem 1. *Let $F'(1) = 1$ and $F''(1) < \infty$. Then*

$$\left(\frac{W_n}{\mathbf{E}W_n}\,;\,\frac{S_n}{\mathbf{E}S_n}\right) \Longrightarrow (W^*;S^*)\,,$$

where

$$\mathbf{E}\left[e^{-\lambda W^* - \theta S^*}\right] = \left[ch\sqrt{\theta} + \frac{\lambda}{2}\frac{sh\sqrt{\theta}}{\sqrt{\theta}}\right]^{-2}, \quad \lambda, \theta > 0.$$

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Karshi State University, 17, Kuchabag street, 180100 Karshi city, Uzbekistan

E-mail address: imomov_ azam@mail.ru, tuhtaev.erkin@mail.ru

MALLIAVIN-TYPE REPRESENTATION FOR THE SENSITIVITY OF THE LIKELIHOOD FUNCTION OF DISCRETELY OBSERVED LEVY DRIVEN SDE'S

D.O. Ivanenko

We consider scalar equation

$$\mathrm{d}\, X_t = a_\theta(X_t)\,\mathrm{d}\, t + \mathrm{d}\, S_t, \tag{1}$$

with symmetric α-stable stochastic process S, $\alpha \in (0,2)$. Let $\left\{X_{hk},\ k = \overline{1,n}\right\}$ be a set of discrete time observations of the solution X to (1). Let $l_n^\theta\,(x_1, \cdots, x_n) = \sum_{k=1}^n \ln p_h^\theta(x_{k-1}, x_k)$ be logarithm likelihood function, where $p_h^\theta(x,y)$ denotes the transition probability density of the Markov process X.

By means of the Malliavin calculus, we provide integral representation for the derivatives of the function l_n^θ. This representation can be used to prove asymptotic properties of the MLE of the parameter θ.

The talk partially contains results obtained in a collaboration with O.M. Kulik.

National Taras Shevchenko University of Kyiv, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: ida@univ.kiev.ua

SEVERAL TWO-BOUNDARY PROBLEMS FOR LÉVY PROCESSES AND SPECIAL CLASS OF SEMI-MARKOV PROCESSES

Tetyana Kadankova

Lévy processes constitute a fundamental class of stochastic processes which possess nice properties and, thus, proved to find a wide range of applications in different areas such as risk theory and finance, physics, biosciences and telecommunications, queueing theory, fragmentation theory etc. For instance, Lévy processes are an excellent tool for modelling stock price behaviour in mathematical finance. On the other hand, the Lévy processes are important from a theoretical point of view, since other classes of stochastic processes, such as semi-Markov processes, semi-martingales are obtained as generalizations of Lévy processes.

In the present talk we address two-sided exit problems and the asymptotic analysis for various two-boundary characteristics of general Lévy processes and the difference of two compound renewal processes. The methodology we use is mainly based on a probabilistic approach, use of one-boundary characteristics of the process and theory of Fredholm equations of the second kind. This approach appears to be quite universal: it works for general Lévy processes, general random walks, and even for certain semi-Markov processes. The solution of most problems is given in the form of Neumann series. For the special case of Lévy processes, the results are given in closed form, namely in terms of the scale function of the process. We also provide some applications in the context of queueing theory.

Vrije Universiteit Brussel, Department of Mathematics, 1050, Brussels Belgium

E-mail address: tetyana.kadankova@vub.ac.be

THE EXPECTED COUPLING TIME FOR TWO INDEPENDENT DISCRETE RENEWAL PROCESSES

N.V. Kartashov, V.V. Golomosiy

Consider two independent sequences $(\theta_{lk}, k \geq 1), l = 1, 2$, of discrete i.i.d. in each sequence random variables. Let

$$g_{lj} = \mathbf{P}(\theta_{l1} = j),\ j \geq 1, \quad \mu_l = \mathbf{E}\theta_{l1}, \quad \mu_{l2} = \mathbf{E}(\theta_{l1})^2,\ l = 1, 2,$$
$$u_{lj} = \sum_{n\geq 0} (g_l)_j^{*n},\ j \geq 0, \quad l = 1, 2,$$

where g^{*n} is the n-th convolution of sequence g.

Define the discrete renewal processes

$$\tau_{lk} = \tau_{l0} + \sum_{j=1}^{k} \theta_{lj},\ k \geq 0, \quad l = 1, 2.$$

Main subject of our study [1] is the coupling moment

$$\zeta \equiv \inf(\tau_{1n} : \exists n, k \geq 0 : \tau_{1n} = \tau_{2k} > 0).$$

Theorem. *Assume that* $g_{11} + g_{21} > 0$ *and* $\mu_{l2} < \infty, l = 1, 2$. *Then*

$$\mathbf{E}(\zeta \mid \tau_{10} = 0, \tau_{20} = 0) = \mu_1\mu_2,$$

and for all $i, j \geq 0$ *with* $(i, j) \neq (0, 0)$

$$\mathbf{E}(\zeta \mid \tau_{10} = i, \tau_{20} = j) \leq \max(i, j) + C_{12}\ \mathbb{1}_{i\neq j} < i + j + C_{12},$$
$$C_{12} = (\mu_{12}/\mu_1 + \mu_{22}/\mu_2 - 2)(1 - (1 - \overline{u}_1)(1 - \overline{u}_2))^{-1},$$
$$\overline{u}_l \geq \exp(\mu_l(1 - g_{l1})^{-1} \ln g_{l1}),\ l = 1, 2.$$

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: nkartashov@skif.com.ua, mailtower@gmail.com

MODELLING MOLECULAR MOTORS WITH GENERALIZED QUASI-BIRTH-AND-DEATH PROCESSES

Peter Keller

Transport Molecules play an important role in cells. They move along tiny street-like structures, so called filaments, in a non deterministic way. An extension of the well known Quasi-Birth-and-Death Process to the whole integers is used to model the linear movement of the motor molecules both in Markovian and Semi-Markovian regime. We a killing, that is stopping the process after a random time. It models the ability of the motor molecules to detach from the filament. As detachment is almost sure the dynamics is transient. For the application several properties are interesting, e.g. the distribution of the last position before detachment, occupation of the filament or the maximal distance from the starting point. These properties are related to an extension of the double sided exponential distribution (Laplace's first error law) to a matrix version. We present some of the results in form of a poster.

University of Potsdam, Department of Mathematics
E-mail address: peter.keller@mpikg.mpg.de

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GENERATOR OF LIMIT APPROXIMATION PROCESS WITH MARKOV SWITCHING

O.I. Kiykovska[1], Ya.M. Chabanyuk[1], O.M. Kinash[2]

The continuous stochastic approximation procedure [1] in ergodic Markov environment in diffusion approximation schema is governed by the stochastic differential equation:

$$du^\varepsilon(t) = a(t)[C(u^\varepsilon(t); x_t^\varepsilon)dt + \varepsilon^{-1}C_0(u^\varepsilon(t); x_t^\varepsilon)dt + \sigma(u^\varepsilon(t); x_t^\varepsilon)dw(t)],\ u^\varepsilon(0) = u_0, \tag{1}$$

where $u \in R^d$, is a random evolution, $x(t), t \geq 0$, a Markov process in the measurable phase states space $(X, \mathbf{X})$, $C(u; x)$, is a regression function, depending on coupled Markov process $u^\varepsilon(t), x_t^\varepsilon = x(t/\varepsilon^2)$, w a Wiener process, depending on time t, and ε a small parameter series [2].

For the generator Q of the Markov process $x(t), t \geq 0$, with stationary distribution $\pi(B), B \in \mathbf{X}$, is defined the potential R_0 [2].

The average regression function is defined by the relation: $C(u) = \int\limits_X \pi(dx)C(u, x)$.

Limit generator of coupled Markov process $u^\varepsilon(t), x_t^\varepsilon, t \geq 0$, in stochastic approximation procedure is presented as follows

$$\mathbf{L}V(u) = a(t)\mathbf{C}(u)V'(u) + 1/2a^2(t)\mathbf{B}(u)V''(u),$$

where V(u) is a Lyapunow function of averaged system $\frac{du(t)}{dt} = \mathbf{C}(u)$,
and $\mathbf{B}(u) = 2\int_X \pi(dx)C_0(x)R_0C_0(x) + \int_X \pi(dx)\sigma^2(u;x)$.

Remark 1. When $C_0(x) \equiv 0, \mathbf{B}(u) \neq 0$ or $C_0(x) \neq 0, \sigma(u;x) \equiv 0$ we get the limit diffusion process $\zeta(t), t \geq 0$. $\zeta(t)$ is the solution of stochastic differential equation

$$d\zeta(t) = a(t)[C(\zeta(t))dt + \sigma(\zeta(t))dw(t)].$$

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[1] Lviv Polytechnic National University, Applied Mathematics, 12 St. Bandera St., Lviv, Ukraine

[2] Ivan Franco National Univercity Lviv, Theoretical and Applied Statistics, 1, Universytetska St., Lviv, Ukraine

E-mail address: olja.kiykovska@gmail.com

PARAMETRIX CONSTRUCTION OF THE TRANSITION PROBABILITY DENSITY OF SOME LÉVY-TYPE PROCESSES

V. Knopova

The talk is devoted to the parametrix construction of the fundamental solution to the equation

$$\frac{\partial}{\partial t}u(t,x) = L(x,D)u(t,x), \quad t > 0, \quad x \in \mathbb{R}, \tag{1}$$

where for $v \in C_0^\infty(\mathbb{R})$

$$L(x,D)v(x) := \int_{\mathbb{R}} (u(x+y) - u(x))m(x,y)\mu(dy),$$

μ is a Lévy measure, and $m(x,y)$ satisfies some mild regularity assumptions. We show that the solution to (1) can be constructed by Levi's parametrix method, and derive the estimates for it. Further, we show that this solution is the transition probability density of a Markov process whose generator is $L(x,D)$.

The talk is based on the on-going research work with Alexei Kulik.

V.M. Glushkov Institute of Cybernetics NAS of Ukraine, Glushkov Ave. 40, Kyiv 03187, Ukraine.

E-mail address: vicknopova@googlemail.com

STORAGE IMPULSIVE PROCESSES ON INCREASING TIME INTERVALS

V.S.Koroliuk, R. Manca, G. D'Amico

The Storage Impulsive Process (SIP) $S(t)$ is a sum of (jointly independent) random variables defined on the embedded Markov chain of a homogeneous Markov process.

The SIP considered in the series scheme on increasing time intervals t/ε with a small parameter $\varepsilon \to 0 (\varepsilon > 0)$. The SIP is investigated in the average and diffusion approximation scheme. The large deviation problem is considered under corresponding scaling with an asymptotically small diffusion.

01601, IM NASU, Tereschekivska str. 3, Kyiv, Ukraine

E-mail address: vskorol@yahoo.com

CONVERGENCE OF THE OPTIMIZATION PROCEDURE WITH SEMI-MARKOV SWITCHING IN ONE-DIMENSIONAL CASE

V.R. Kukurba, Ya.M. Chabanyuk

Continuous of the stochastic optimization procedure with semi-Markov switching is defined by the evolution equation

$$du^\varepsilon(t) = a(t)\nabla_b C(u^\varepsilon(t); x(t/\varepsilon)), t \geqslant 0, u^\varepsilon(0) = u_0, \tag{1}$$

where $u \in R$, $\nabla_{b(t)}C(u;x) = (C(u+b(t);x) - C(u-b(t);x))/2b(t)$,$a(t) > 0$,$b(t) > 0$, and regression function $C(u;x)$ depends on uniformly ergodic semi-Markov process $x(t) > 0, t \geqslant 0$, in the dimensional phase space of states $(X, \mathbf{X})$, that is defined by kernel $Q(x,B,t) = P(x,B)G_x(t)$, where $G_x(t)$- distribution function of time spent in the current state $x \in X$. Associated Markov process $x_0(t), t \geqslant 0$ is defined by the generator $\mathbf{Q}$[1]. Stochastic optimization procedure is characterized by the extended compensating operator $\mathbf{L}_t$[2].

Theorem 1. *Let the Lyapunov function be $V(u), u \in R$, that ensures exponential stability of the averaged system [2] $C'(u)V'(u) < -c_0V(u), c_0 > 0$.*

Also $\tilde{\nabla}_{b(t)}C(u;x) := \nabla_{b(t)}C(u;x) - \nabla_{b(t)}C(u)$, there are additional conditions: $V'(u) \leqslant c_1(1+V(u))$, $|\nabla_{b(t)}C(u) - C'(u)| \leqslant k_1b(t)$, $|\nabla_{b(t+\varepsilon s)}C(u;x)V'(u)| \leqslant c_3(1+V(u))$, $|\nabla_{b(t+\varepsilon s)}C(u;x)[\nabla_{b(t+\varepsilon s)}C(u;x)V'(u)]'| \leqslant c_4(1+V(u))$,

$|\nabla_{b(t+\varepsilon s)}C(u;x)\mathbf{PR}_0[\tilde{\nabla}_{b(t)}C(u;x)V'(u)]'| \leqslant c_5(1+V(u))$.

Distribution function $G_x(t)$,$x \in X$, uniformly satisfy the Cramer's condition, evenly $x \in X$: $\sup\limits_{x\in X} \int_0^\infty e^{ht}\bar{G}_x(t)dt \leqslant H < \infty, h > 0$.

Finally, functions $a(t)$ and $b(t)$ monotonously decreasing, limited and satisfy the conditions: $\int_0^\infty a(t)dt = \infty, \int_0^\infty a(t)b(t)dt < \infty \int_0^\infty a^2(t)dt < \infty$ $\frac{a'(t+\varepsilon s)}{a^2(t)a(t+\varepsilon s)} \leqslant A_1$, $\frac{b'(t+\varepsilon s)}{a^2(t)b'(t+\varepsilon s)} \leqslant A_2$, $\frac{a(t+\varepsilon s)b'(t+\varepsilon s)}{2a^2(t)b(t+\varepsilon s)} \leqslant A_3$, $a(t) > 0, b(t) > 0$, $0 \leqslant t, \varepsilon > 0$. Then for each positive $\varepsilon \leqslant \varepsilon_0, \varepsilon_0$ - is sufficiently small, stochastic optimization procedure (1) converges with $t \to \infty$ with probability 1 to the extremum point of averaged system:

$P\{\lim\limits_{t\to\infty} u^\varepsilon(t) = 0\} = 1$.

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Lviv Polytechnic National Univercity, Applied Mathematics, 12 St. Bandera St., Lviv, Ukraine

E-mail address: vkuku@i.ua

FRACTIONAL PEARSON DIFFUSION

Nikolai N. Leonenko

This is a joint work with Mark M. Meerschaert and Alla Sikorskii (Michigan State University).

Pearson diffusions have stationary distributions of Pearson type. They includes Ornstein-Uhlenbeck, Cox-Ingersoll-Ross, and several others well-kown processes. Their stationary distributions solve the Pearson equation, developed by Pearson in 1914 to unify some important classes of distributions (e.g., normal, gamma, beta). Their eigenfunction expansions involve the traditional classes of orthogonal polynomials (e.g., Hermite, Laguerre, Jacobi). We develop fractional Pearson diffisions, constracting by a non-Markovian inverse stable time change. Their transition densities are shown to solve a time-fractional analogue to the diffusion equation with polinomial coefficients. Because this process is not Markovian, the stochastic solution provides additional information about the movement of particles that diffuse under this model.

Anomalous diffusions have proven useful in applications to physics, geophysics, chemistry, and finance.

Cardiff University, Cardiff, United Kingdom

E-mail address: leonenkon@cardiff.ac.uk

THE LAPLACE-STIELTJES TRANSFORM OF JOINT DISTRIBUTION OF FIRST CROSSING SOME LEVEL AND ACROSS OVER THIS LEVEL WITH DIFFERENTIATED AND DELAYING SCREEN SEMI-MARKOV PROCESS

T.H. Nasirova[1], M.H. Mikaylov[2], U.C. Idrisova[2]

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1. Introduction. To find the distribution of the Semi-Markov random walk and its main boundary functional some authors use the asymptotic, factorization and etc. methods ([1], [2], [3], etc.). In this paper we tapered the class of the distribution of the walk and finded the Laplace-Stieltjes transform of joint distribution of first crossing some level and across over this level with differentiated and delaying screen Semi-Markov process.

2. Mathematical Statement of the problem. Let on the probability space $(\Omega, F, P(\bullet))$ is given the sequence $\{\xi_k^+, \eta_k^+, \xi_k^-, \eta_k^-\}_{k=\overline{1,\infty}}$ of independent equal distributed positive and between themselves independent random variables $\xi_k^+, \eta_k^+, \xi_k^-, \eta_k^-$, $k = \overline{1,\infty}$.

Using these random variables we can construct the following processes $X^\pm(t) = \sum\limits_{i=1}^{k-1} \eta_i^\pm$ if $\sum\limits_{i=1}^{k-1} \xi_i^\pm \leqslant t < \sum\limits_{i=1}^{k} \xi_i^\pm$, $k = \overline{1,\infty}$.

The process $X_1(t) = X^+(t) - X^-(t)$ we shall call the process with differentiated random walk.

We denote $\tau_k^\pm = \sum\limits_{i=1}^{k} \xi_i^\pm$, $k = \overline{1,\infty}$.

We put the random variables in increasing order. We denote obtained sequence by $\{\tau_k\}$, $k = \overline{1,\infty}$.

Let $\eta_k = \begin{cases} \eta_i^+, & \text{if} \quad \tau_k = \tau_i^+, \\ \eta_j^-, & \text{if} \quad \tau_k = \tau_j^-. \end{cases}$

We construct the following process

$X(t) = \varsigma_k$, if $\tau_k \leqslant t < \tau_{k+1}$, $k = \overline{1,\infty}$, where $\varsigma_o = z$, $\varsigma_k = \max(0, \varsigma_{k-1} + \eta_k)$, $z > 0$.

We shall call this process – the Semi-Markov process with differentiated random walk and delaying screen at zero.

Our aim is to find the Laplace-Stieltjes transform of joint distribution of first crossing some level and across over this level with differentiated and delaying screen Semi-Markov process.

We denote

$$\tau_1^a = \inf\{t : X(t) \geqslant a\}, \quad \gamma_1^a = X(\tau_1^a + 0) - a,$$

$$K(t, \gamma|z) = P\{\tau_1^a < t; \gamma_1^a > \gamma|X(0) = z\}t > 0, \;\; \gamma > 0.$$

$$\tilde{K}(\theta, \gamma|z) = \int\limits_{t=0}^{\infty} e^{-\theta t} K(t, \gamma|z)dt, \;\; \theta > 0, \quad \tilde{\tilde{K}}(\theta, \delta|z) = \int\limits_{t=0}^{\infty} e^{-\delta t} d_\gamma \tilde{K}(\theta, \gamma|z)dt, \;\; \delta > 0.$$

It was found the integral equation for $\tilde{\tilde{K}}(\theta, \delta|z)$ which following solution

$$\tilde{\tilde{K}}(\theta, \delta|z) = c_1(\theta)e^{K_1(\theta)z} + c_2(\theta)e^{K_2(\theta)z} +$$

$$+ \frac{(\mu_- - \delta)}{\lambda_+ + \lambda_- + \theta} e^{\delta\, a} \frac{[(\lambda_- + \theta)\mu_+ + (\lambda_+ + \lambda_- + \theta)\delta]}{[\theta + K_1(\theta)][\theta + K_2(\theta)]} e^{-\delta\, z},$$

where $k_1(\theta), k_2(\theta)$ are the roots of characteristic equation, corresponding differential equation which is obtained from integral equation for $\tilde{\tilde{K}}(\theta, \delta|z)$, $c_1(\theta), c_2(\theta)$ finding from boundary conditions.

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[1] Baku State University Azerbaijan
[2] Institute of Cybernetics of ANAS. Baku, Azerbaijan F.Agaev st., 9, AZ1141, Baku, Azerbaijan
E-mail address: ulviyyeidrisova@yahoo.com

APPLICATION OF SEMI-MARKOV PROCESSES FOR SYSTEMS MODELLING

Yu.E. Obzherin, A.I. Peschansky

Today, the theory of Semi-Markov processes with a common phase field of states [1, 2] is widely applied for different systems modelling.

To develop this approach the authors consider the possibility of this class of random processes application to modelling:
- queueing systems;
- systems with time reservation;
- systems with maintenance;
- systems with control execution.

For the mentioned systems Semi-Markov models with a discrete-continuous phase field of states have been built. The stationary characteristics have been defined. The results of application of phase merging algorithms are reported. The definition of stationary characteristics enabled to solve the tasks of optimization of control and maintenance execution period.

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Sevastopol National Technical University, Sevastopol, Ukraine
E-mail address: vmsevntu@mail.ru

VARIABLE LENGTH MARKOV CHAINS AND PIECEWISE DETERMINISTIC MARKOV PROCESSES

P. Vallois[1], P. Cenac[2], B. Chauvin[3], S. Herrmann[4]

We are interested in Markov chains (Y_k, M_k) where Y_k takes its values in a finite set and M_k denotes the last sojourn time of Y_i at Y_k and up to time k. Thus, Y_k is Variable Length Markov Chain and M_k represents the variable memory. Under certain conditions, we prove that the Markov chain (Y_k, M_k) has a unique invariant probability measure and we also give a path description of (Y_k, M_k). Moreover we explicit a link between some context trees, in particular the simple and the double infinite comb.

In the case case where Y_k is valued in $\{-1, 1\}$, we consider the random walk (S_k) associated with the above Markov chain, i.e. the process whose increments are the Y_k's. It can be proved that there exists a normalization of (S_k) such that it converges in distribution to a Piecewise Deterministic Markov Process. Moreover, the trajectory of the limit process is a succession of straight lines with slope ± 1 and random length.

[1] Université de Lorraine, Institut Elie Cartan, UMR 7502 et Projet INRIA BIGS, B.P. 239, F54506 Vandoeuvre les Nancy Cedex *E-mail address*: vallois@univ-lorraine.fr
[2] Université de Bourgogne
[3] Université de Versailles
[4] Université de Bourgogne

INVARIANT FINITELY ADDITIVE MEASURES OF ONE OSCILLATING RANDOM WALK

Alexander Zhdanok

We consider one oscillating random walk $\{x_n, n \geqslant 0\}$ with irrational steps

$$x_{n+1} = \{x_n + \xi_n, \text{if } x_n < 0; x_n + \eta_n, \text{if } x_n > 1; x_n + (1 - x_n)\xi_n + x_n\eta_n, \text{if } 0 \leqslant x_n \leqslant 1\}$$

where ξ_n, η_n - independent random variables taking on two equiprobable values $P\{\xi_n = -1\} = P\{\xi_n = +\sqrt{2}\} = 0,5$, $P\{\eta_n = +1\} = P\{\eta_n = -\sqrt{3}\} = 0,5$. This walk generates the oscillating Markov chain (OMC) with a Feller property.

Studying of similar to these non-lattice walks were held by A.A. Borovkov. In [1] he proved the existence and uniqueness of invariant *countably additive* probability measure π for the OMC, where $\pi((a,b)) > 0$ for each interval.

In the present work we announce some ergodic properties of this OMC obtained through course developed by the author, an *finitely additive approach* in the operator theory of Markov chains (see,eg, [2]). We divide the phase space of the OMC, i.e. number line $\mathbb{R}$, the continuum minimal stochastically closed "trajectory" sets $M_\xi = \{\xi + m + s\sqrt{2} + t\sqrt{3} | m, s, t \in \mathbb{Z}\}$.

It can be shown, that if the initial point $x_0 \in M_\xi$, then there is no invariant *countably additive* probability measures on M_ξ.

Theorem 1. *For the OMC on any M_ξ there exist is a probability invariant purely finitely additive measure μ_ξ, with the property $\mu_\xi((a,b) \cap M_\xi) > 0$ for any interval.*

Theorem 2. *Countably additive measure π, invariant for the OMC "on the whole $\mathbb{R}$", is regularization of the all invariant purely finitely additive measures μ_ξ on M_ξ.*

We note the following some version of "**zero-one law**." We divide the family of all trajectory sets M_ξ, on any two *measurable* disjoint parts R_1 and R_2.

Theorem 3. *For any probability finitely additive invariant measure μ for the OMC, including for a countably additive π, either $\mu(R_1) = 0$, or $\mu(R_1) = 1$ is performed.*

We also consider certain questions of convergence OMC to invariant measures.

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Tuva State University, Russia, Tuva, Kyzyl
E-mail address: zhdanok@inbox.ru

GENERALIZATION OF OPTIONAL DECOMPOSITION THEOREM

N.S. Gonchar

The optional decomposition theorem was first given in paper of El Karoui and Quenez in [1] for special class of model and in general supermartingale context proved by Kramkov [2] under assumptions that evolution of risk actives is locally bounded and supermartingale is nonnegative. Later Fölmer and Kabanov [3] proved it in full generality. Optional decomposition arises in context of incomplete financial markets.

Our approach to the solution of supermartingale decomposition problem is general and is the following: what family of measures have to be that supermartingale f_t to admit decomposition

$$f_t = M_t - g_t \tag{1}$$

where M_t is a martingale with respect to all measures from M, g_t is increasing process.

Theorem 1. *Let M be a family of equivalent measures on the right continuous stochastic basis $\{\Omega, \Im_t, \Im\}$ and φ be a nonnegative random variable such that $E^P\varphi < T < \infty$, $P \in M$. If f_t is a right continuous supermartingale with respect to the family M, satisfying condition $|f_t| \leqslant \varphi$, then there exists right continuous martingale M_t with respect to any measure from M and $\Im_t$ adapted increasing process g_t such that () takes place, under condition that there exists a countable subset in M dense in it with respect to metrics $|P_1 - P_2| = \sup|f_1 - f_2|$, where Q is fixed measure in M, $P_i(A) = \int_A f_i dQ$, $i = 1, 2$.*

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Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
E-mail address: honchar@i.ua

NON HOMOGENEOUS MARKOV CHAIN DESCRIBING BANK WORKING

N.S. Gonchar[1], D.A. Kaplunenko[2]

New stochastic model of bank work was earlier proposed and investigated in [1,2]. Here we consider stochastic non-homogenous Markov chain for description of evolution of bank capital R_n, $n = 0, 1, 2, ...$ Suppose that $R_n = f(n, R_{n-1}, \zeta_n)$, where $\zeta_n, n = 0, 1, 2, ...$ is a sequence of independent random variables describing risk of investment of bank capital in risk assets and taking into account receipt of deposits to accounts of the bank, fulfillment of liabilities and so on. Denote by $P(k-1, y_{k-1}, k, A) = E\chi_A(f(k, y_{k-1}, \zeta_k))$ transition probability function for one step in considered model.

Theorem 1. *Let $\psi_k(y_k), k = 0, 1, 2, ...$ be the probability to crash after time moment $t = k$ with capital $y_k \geqslant 0, k = 0, 1, 2, ..$ Then the vector $\psi = \{\psi_k(y_k)\}_{k=0}^{\infty}$ satisfies the set of equations*

$$\psi_k(y_k) = 1 - P(k, y_k, k+1, [0, \infty)) + \int_0^\infty P(k, y_k, k+1, dy_{k+1})\psi_{k+1}(y_{k+1}), \ k = 0, 1, 2, ...$$

having solution given by the series $\psi = \sum_{m=0}^{\infty} k^m \varphi_0$, where $(kf)_k(y_k) = \int_0^\infty P(k, y_k, k+1, dy_{k+1}) f_{k+1}(y_{k+1})$, $f = \{f_0(x), f_1(y_1), ..., f_n(y_n), ..\}$, $\varphi_0 = \{\varphi_0(y_0), ...\varphi_k(y_k), ..\}$, $\varphi_k(y_k) = 1 - P(k, y_k, k+1, [0, \infty))$.

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[1] Bogolyubov Institute for Theoretical Physics of the National Academy of Sciences of Ukraine
E-mail address: mhonchar@i.ua
[2] Taras Shevchenko Kyiv University, Kyiv, Ukraine
E-mail address: darya_mk@bigmir.net

SOME FUNCTIONAL ANALYTIC PROBLEMS CONNECTED TO THE UTILITY MAXIMIZATION PROBLEM

A.A. Gushchin

Utility maximization problem is a classical problem in mathematical finance. From mathematical point of view, it is a class of problems dealing with an optimal control of stochastic processes. To solve it, dual methods are often used. In the talk we shall discuss how to state a dual problem and to establish dual relations. Such a problem can be formulated in abstract terms and refers essentially to functional analysis. Schematically, the solution of the problem consists of three steps: 1) to reduce an original problem to a similar one in appropriate functional spaces, e.g. L^∞ (maybe, with a weight function), or Orlicz spaces with respect to a family of measures; 2) to state a dual problem using duality theorems; 3) to get rid of singular functionals in the dual problem.

The talk is based on the papers [1]–[3].

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Steklov Mathematical Institute, Gubkina 8, 119991 Moscow, Russia

E-mail address: gushchin@mi.ras.ru

APPLICATION OF FRACTIONAL STABLE LEVY PROCESSES FOR MODELING FINANCIAL TIME SERIES

L.O. Kirichenko, V.V. Kyriy, A.V. Storozhenko

The new directions of investigation of financial series are based on stable random processes possessing the properties of self-similarity and heavy tails. A random variable X is called α-stable, if for every two positive numbers a and b we can find $c > 0$, that $Law(aX_1 + bX_2) = Law(cX)$. In this case exists such value $\alpha \in (0; 2]$, that the equality $c^\alpha = a^\alpha + b^\alpha$ is executed. α is called the index of stability or the tail index. A stochastic process is Hurst process with a self-similar parameter H, $(0 < H < 1)$, if it satisfies: $Law\{X(t)\} = Law\{a^{-H}X(at)\}$, $\forall a > 0, t > 0$. A process $L_\alpha(t)$ is $\alpha-$stable Levi motion, if it possesses stationary independent $\alpha-$stable increments. This process is self-similarly with parameter $H = 1/\alpha$. Fractional stable Levy motion $L_{\alpha,H}(t)$ is self-similarly and stable process with stationary increments and infinity correlation interval: $L_{\alpha,H}(t) = \frac{1}{\Gamma(H+1/2)} \int_0^t (t-\tau)^{H-1/2} dL_\alpha(\tau)$. Fractional Levy motion is a generalization of fractional Brownian motion, where increments are $\alpha-$stable random variables. $L_{\alpha,H}(t)$ is self-similarly process: $Law\left\{L_{\alpha,H}(ct_2) - L_{\alpha,H}(ct_1)\right\} = Law\left\{c^{H-1/2+1/\alpha}\left(L_{\alpha,H}(t_2) - L_{\alpha,H}(t_1)\right)\right\}$.

In this work we have carried out the estimation of stable distribution parameters and Hurst exponent of financial series and series of economical indicators. It's showed that models of such series can be fractional stable Levy processes. We generate realizations of the fractional stable Levy process by using Riemann-sum approximations of its stochastic integral using the fast Fourier transform algorithm.

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Kharkov National University of Radio Electronics, Department of Applied Mathematics and Management, Lenin av. 14, Kharkov, 61166, Ukraine

E-mail address: ludmila.kirichenko@gmail.com

ARBITRAGE ABSENCE IN ECONOMY DYNAMICAL SYSTEMS WITH FIXED GAINS

S.V. Kuchuk-Iatsenko

This paper is devoted to construction of non-arbitrage economy dynamics for the case when economy agent's strategy does not depend on price vector. The case of proportional consumption with fixed gains is treated.

The economy system functions during N periods, $N < \infty$. In the t-th period of economy operation demand vectors $C_i^t(\omega) = \{C_{ki}^t(\omega)\}_{k=1}^n$, $i = \overline{1,l}$ are given on a probability space $\{\Omega, F, P\}$. Consumption in the t-th period is characterized by some random vector $y^t(\omega) = \{y_i^t(\omega)\}_{i=1}^l$, $y_i^t(\omega) > 0$. the supply vector in the t-th period is given by the formula $\psi^t(\omega) = \sum_{i=1}^l C_i^t(\omega) y_i^t(\omega)$.

Let vectors $C_i^t(\omega)$, $i=\overline{1,l}$, satisfy inequalities

$$\sum_{s=1}^{l} C_{ks}^t(\omega)>0, \quad k=\overline{1,n}, \quad \sum_{k=1}^{n} C_{ki}^t(\omega)>0, \quad i=\overline{1,l}, \quad t=\overline{1,N},$$

and vectors $y^t(\omega)=\{y_i^t(\omega)\}_{i=1}^{l}$ are strictly positive for each $\omega\in\Omega$.

Theorem 1. *The economy dynamics described above is non-arbitrage if the following inequalities hold with probability 1:*

$$\bar{p}_k^{t+1}\le \bar{p}_k^t\left(1+f^{t+1}\left(\bar{p}_1^{t+1}\right)\right), \quad k=\overline{2,n_t}, \quad t=\overline{1,N},$$

where $\bar{p}^t=\{\bar{p}_k^t\}_{k=1}^{n_t}$ - equilibrium price vector in the t-th period of economy operation, $f^t(x),\ t=\overline{1,N}$, - is the set of nonnegative functions which satisfy the condition $0\le f^t(x)\le x,\ t=\overline{1,N}$.

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Taras Shevchenko Kyiv University, Kyiv, Ukraine

E-mail address: kuchuk.iatsenko@gmail.com

STATISTICAL ESTIMATES IN THE MODELS WITH LONG-RANGE DEPENDENCE

Yu.S. Mishura

We consider a stochastic differential equation involving standard and fractional Brownian motion with unknown drift parameter to be estimated. We investigate the standard maximum likelihood estimate of the drift parameter, two non-standard estimates and three estimates for the sequential estimation. Model strong consistency and some other properties are proved. The linear model and Ornstein-Uhlenbeck model are studied in detail. As an auxiliary result, an asymptotic behavior of the fractional derivative of the fractional Brownian motion is established. This is joint work with Alexander Melnikov and Yurij Kozachenko.

01601 Ukraine Kyiv Volodymyrska 60 National Taras Shevchenko University of Kyiv

E-mail address: myus@univ.kiev.ua

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ON EQUIVALENT MARTINGALE MEASURES

E.D. Patkin

Let the evolution of risk assets be gives by the law: $S_n=(1+\rho_n)S_{n-1}$, $n=1,...,N$, where ρ_n - the sequence of independent identically distributed random variables, taking values on $(-1,+\infty)$. The problem is to describe the set of all martingale measures, provided ρ_n is defined on $\{\Omega,\ F,\ P_0\}$, and non-risk asset $B_1=1$.

Theorem 1. *The set of equivalent martingale measures is nonempty, if: 1) $E^{P_0}g_n(\rho_n)\rho_n=0$; 2) $E^{P_0}g_n(\rho_n)=1$. And there exists the sequence of Borel functions $g_n(x)$ such that condition 1) and the following condition 2) are fullfilled: 2) $0\leqslant g_n(x)<\infty$, $-1<x\leqslant c$, $c<\infty$ and $P(g_n(\rho_n)=0)=0\,\forall n=1,...,N$, where $P(A)=\int_A\prod_{n=1}^{N}g_n(\rho_n)dP$.*

All other martingale measures are described as follows: $Q(A)=\int_A f(\omega)dP_0$, $f(\omega)\geqslant 0$, $P_0(f(\omega)=0)=0$, $E^{P_0}f(\omega)=1$, $E^{Q}\rho_n=0$, $n=1,...,N$.

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Taras Shevchenko National University, Kyiv, Ukraine

E-mail address: Patkin88@mail.ru

MONTE CARLO SIMULATION METHODS FOR RARE EVENTS

M.S. Pupashenko

A rare event is an event occurring with very small probability, where the definition of "small" depends on the application domain. Although rare events usually happen seldom, i.e. with small probability, they are indeed very important in many application areas. Therefore, an accurate estimation of the probabilities of such events is important.

We concentrate on the methods of value-at-risk (VaR) estimation. The basic Monte Carlo method which would be the first idea to estimate VaR involves a great number of portfolio revaluations which are usually time costly. Therefore one should apply a variance reduction technique in order to reduce the number of portfolio revaluations.

The main results in the area of estimating the VaR using the Monte Carlo simulation were achieved in [3], [4], and summarized in [2]. We are trying to improve those methods by using the most recent results regarding importance sampling from [1] and [5] and by applying new iterative techniques.

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University of Kaiserslautern, Department of Mathematics, Erwin-Schroedinger-Strasse, Geb. 48, 67653, Kaiserslautern
E-mail address: myhailo.pupashenko@gmail.com

PROBLEMS OF RESELLING THE OPTIONS

M.S. Pupashenko

We consider the problem of optimal reselling the European option in case of implied volatility being the Cox-Ingersoll-Ross process. In paper Pupashenko and Kukush (2008) arbitrage free discrete approximation was proposed and the optimal Investor strategies were described by nonrandom stopping sets in the phase space of possible implied volatility and stock price.

Here we are interested in the expected reward functional of the form

$$\Phi(\mathcal{M}_T) = \sup_{\tau \in \mathcal{M}_T} \mathsf{E} e^{-r\tau} C(\tau, S(\tau), \sigma(\tau)),$$

where $C(t, S(t), \sigma(t))$ is a price of an option and $\mathcal{M}_T$ is a class of stopping times $0 \le \tau \le T$ with respect to filtration $\mathcal{F}_t = \sigma((S(s), \sigma(s)), s \le t)$ generated by the vector process $(S(t), \sigma(t))$ of asset prices and implied volatility. We have constructed a two-dimensional binomial-trinomial exponential approximation and proved that it is arbitrage-free. In order to prove the convergence of the optimal expected reward in the model to the optimal expected reward functional for the corresponding two-dimensional exponential process with independent log-increments we modified the Cox-Ingersoll-Ross model for implied volatility in a way that it keeps its main properties and satisfies all necessary conditions from paper Lundgren et al. (2008).

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University of Kaiserslautern, Department of Mathematics, Erwin-Schroedinger-Strasse, Geb. 48, 67653, Kaiserslautern
E-mail address: myhailo.pupashenko@gmail.com

MINIMAX ESTIMATING FOR RISKY ASSET MODELS

N.U. Shchestyuk

The Bkack–Shcoles model gives options prices as a function of volatility. If an option price is given by the market we can invert this relationship to get the implied volatility. If the model were perfect, this implied value would be the same for all option market prices, but reality shows this is not the case. Implied BlackвЂ"Scholes volatilities strongly depend on the maturity T and the strike K of the European option. It is easy to solve this paradox by allowing volatility to be time-dependent, as Merton did (see Merton, 1973). But the dependence of implied volatility on the strike K, for a given maturity T (known as the smile effect) is trickier. The spot strike process $S(t)$ is then described by the stochastic differential equation Dupire.

In this paper we consider another approach to estimate historical volatility $\sigma(t) + \epsilon(t)$ and implied volatility $\sigma(T, K) + \epsilon(T, K)$. The various classes of correlation uncertainty for volatility $\sigma(z)$, $z \in D \subset R^2$ are considered. The least favourable spectral densities and the minimax-robust spectral characteristics of the optimal linear estimates of the linear functional $A_\sigma(z) = \sum a(z)\sigma(z)$, $z \in D \subset R^2$ are found for various classes of random fields $\sigma(z), \epsilon(z)$.

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National university "Kijyv–Mohila Academy", Ukraine
E-mail address: Natalyshch@gmail.com

ABOUT THE EQUALITY OF SOLUTIONS OF CAPITAL PRESERVATION PROBLEM AND RESPECTIVE MATRIX GAME UNDER DEVIATION OF INITIAL DATA

Ya.I. Yelejko, K.V. Kosarevych

The capital preservation problem is considered as pair zero-sum R-matrix game. It is generally believed that solution of capital preservation problem is the same as solution of respective matrix game. However, this result is only proved when optimal strategy of second player (external environment) is completely mixed. Therefore, the question under deviation of this condition is opened.

Let (P^*, Q^*) be solution in mixed strategies of R-matrix game and suppose that optimal strategy Q^* of second player is not completely mixed. Let the family of $m \times n$ matrices $\{R^\varepsilon\}$, such that $R^\varepsilon \to R$, $\varepsilon \to 0$, exist, and assume strategy $Q^{\varepsilon *}$ is completely mixed for R^ε-matrix game.

Required conditions to $P^{\varepsilon *} = X^{\varepsilon *}$, $\forall \varepsilon$, are derived ($P^{\varepsilon *}$ is an optimal strategy of first player for R^ε-matrix game, $X^{\varepsilon *}$ is a solution of capital preservation problem $D(R^\varepsilon_\Pi) \to \min\limits_{X^\varepsilon}$).

We also show that $X^{\varepsilon *} \to X^*$, $\varepsilon \to 0$, where X^* is a solution of capital preservation problem $D(R_\Pi) \to \min\limits_X$. Finally, we find out $P^{\varepsilon *} \to P^*$, $\varepsilon \to 0$, where P^* is a solution of R-matrix game. Based on the findings of the study, the following conclusion is drawn: $P^* = X^*$.

Ivan Franko National University of L'viv, Department of Theoretical and Applied Statistics, Lviv, Universytetska, 1, 79000
E-mail address: yikts@yahoo.com, katia_kosarevych@mail.ru

SOME NEW RESULTS ON ARBITRAGE IN FRACTIONAL PRICING MODELS

Esko Valkeila

We work with fractional Brownian motion B with Hurst index $H > \frac{1}{2}$. We show that a path-wise stochastic integral with respect to B can have any distribution, and that any random variable $\chi \in F_1^B$ measurable with the filtration of B has a path-wise integral representation with respect to B in an improper sense. We indicate how these results can be applied to obtain new arbitrage examples in the fractional Black-Scholes market model.

The talk is based on joint work with Y. Mishura and G. Shevchenko [2].

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Department of Mathematics and Systems Analysis Aalto University, Finland
E-mail address: esko.valkeila@aalto.fi

STOCHASTIC MODEL OF INTERBRANCHED BALANCE. THE LIMITED THEOREM

Zolotaya A.V., Shevlyakov A.Y.

Stochastic models of Leontiev have been studied in different approaches by the authors [1]-[3].

Let on the probability space $\{\Omega, F, P\}$ be given two sequences $\xi_1, ..., \xi_n$ and $\eta_1, ..., \eta_n$, which are independent from one another and the elements of each sequence are independent having density function

$f_\xi(x) = \begin{cases} n^{1/2}, x \in [0, n^{-1/2}] \\ 0, x \notin [0, n^{-1/2}], \end{cases}$ $\quad f_\eta(x) = \begin{cases} 2nx, x \in [0, n^{-1/2}] \\ 0, x \notin [0, n^{-1/2}]. \end{cases}$

Let's consider the following Leontiev model

$$x_i - \sum_{j=1}^{n} \xi_i \xi_j x_j = \eta_i, i = \overline{1, n}.$$

We denote $\lambda_n = \sum\limits_{j=1}^{n} \xi_j^2$, $\mu_n = \sum\limits_{j=1}^{n} \eta_j^2$ and $\nu_n = \sum\limits_{j=1}^{n} \xi_j \eta_j$.

Theorem 1. *Maximum eigenvalue of technology matrix of Leontiev model equals to λ_n, $P\{\lambda_n < 1\} = 1$, that is the model is productive and*

$\lim\limits_{n \to +\infty} \lambda_n = \frac{1}{3}$, $\lim\limits_{n \to +\infty} \mu_n = \frac{1}{2}$, $\lim\limits_{n \to +\infty} \nu_n = \frac{1}{3}$ *and* $\lim\limits_{n \to +\infty} \sum\limits_{j=1}^{n} x_j^2 = 1$,

where the limits are understand as limits according to probability.

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Donetsk National University, Donetsk, Ukraine
E-mail address: a.zolotaya@mail.donnu.edu.ua, allashev@ua.fm

OPTIMIZATION METHODS IN PROBABILITY

MINIMAX ESTIMATION PROBLEMS FOR PERIODICALLY CORRELATED STOCHASTIC PROCESSES

I.I. Dubovetska, M.P. Moklyachuk

A Bochner square integrable over each compact interval function $\zeta : \mathbb{R} \to H = L_2(\Omega, \mathcal{F}, \mathbb{P})$ is called periodically correlated (cyclostationary) with period T stochastic process if $\mathbb{E}\zeta(t) = 0$, $\mathbb{E}\zeta(t)\zeta(s) = \mathbb{E}\zeta(t+T)\zeta(s+T)$ for all $s, t \in \mathbb{R}$. A periodically correlated process $\zeta(t)$ can be represented in the form $\zeta(t) = \sum_{k\in\mathbb{Z}} e^{2\pi ikt/T}\xi_k(t)$, where $\xi_k(t), k \in \mathbb{Z}, t \in \mathbb{R}$ are $L_2([0,T); H)$-valued stationary stochastic processes determined by formula

$$\xi_k(t)(u) = \frac{1}{T}\int_{-T}^{0} e^{-2\pi ik(t+s+u)/T}\zeta(t+s+u)ds.$$

We consider the problem of the mean-square optimal linear estimation of the functional

$$A\zeta = \int_0^\infty a(t)\zeta(t)dt = \sum_{k\in\mathbb{Z}}\int_0^\infty a_k(t)\xi_k(t)dt, \quad a_k(t) = e^{2\pi ikt/T}a(t),$$

that depends on the unknown values of the process $\zeta(t)$ from observations of the process $\zeta(t)$ for $t < 0$. Formulas are proposed for computing the value of the mean-square error and the spectral characteristic of the optimal linear estimate of the functional $A\zeta$ in the case where spectral density of the process is known. The least favorable spectral densities and the minimax (robust) spectral characteristics of the optimal estimates of the functional $A\zeta$ are determined in the case where the spectral density is not known, but a class of admissible densities is given.

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Kyiv National Taras Shevchenko University, The Faculty of Mechanics and Mathematics, Volodymyrska 64, Kyiv, 01601, Ukraine

E-mail address: `idubovetska@gmail.com`

ON THE METHOD OF EMPIRICAL AVERAGES IN A STOCHASTIC PROGRAMMING PROBLEM

P.S. Knopov, G.D. Bila

We consider the following stochastic programming problem:

$$F(x) = Ef(x,\xi) \to \min_{x\in K},$$

where $x \in K$, and K is a compact set in some topological space. This problem is approximated by the following one:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} f(x,\xi_i) \to \min_{x\in K}.$$

Assume that $x^* \in arg\min F(x)$, $x_n \in arg\min F_n(x)$. We establish the conditions under which

$$x_n \to x^*, \quad F(x_n) \to F(x^*) \quad \text{as } n \to \infty \quad \text{with probability 1.}$$

Similar problems are considered in [1, 2].

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V.M. Glushkov Institute of Cybernetics of NAS of Ukraine, Glushkov Avenue 40, Kyiv 03187, Ukraine.

E-mail address: `knopov1@yahoo.com, chesakg@mail.ru`

MINIMAX INTERPOLATION PROBLEM FOR STOCHASTIC SEQUENCES WITH STATIONARY INCREMENTS

M.M. Luz, M.P. Moklyachuk

The problem of optimal linear estimation of the functional $A_N\xi=\sum_{k=0}^{N}a(k)\xi(k)$ which depends on the unknown values of stochastic sequence $\xi(k)$ with n-th stationary increments with step μ based on observations $\{\xi(m):m\in\mathbb{Z}\setminus\{0,1,\dots,N\}\}$ is considered. Formulas for calculation the mean square error and the spectral characteristic of the optimal linear estimate of the functional $A_N\xi$ are derived in the case where the spectral density $f(\lambda)$ of the sequence is known. In this case

$$h_{\mu}^{(a)}(\lambda)=A_{\mu,N}(e^{i\lambda})(1-e^{-i\lambda\mu})^n\frac{1}{(i\lambda)^n}-\frac{(-i\lambda)^nC_{\mu,N}(e^{i\lambda})}{(1-e^{i\lambda\mu})^nf(\lambda)},$$

$$A_{\mu,N}(e^{i\lambda})=\sum_{k=0}^{N}(D_N^{\mu}a)_ke^{i\lambda k},\quad C_{\mu,N}(e^{i\lambda})=\sum_{k=0}^{N+\mu n}(F^{-1}[D_N^{\mu}a]_{+\mu n})_ke^{i\lambda k},$$

$$\Delta(f)=\int_{-\pi}^{\pi}\frac{\lambda^{2n}\left|\sum_{k=0}^{N+\mu n}(F^{-1}[D_N^{\mu}a]_{+\mu n})_ke^{i\lambda k}\right|^2}{|1-e^{i\lambda\mu}|^{2n}f(\lambda)}d\lambda=\langle F^{-1}[D_N^{\mu}a]_{+\mu n},[D_N^{\mu}a]_{+\mu n}\rangle.$$

Matrices D_N^{μ} and F are determined by coefficients $a(0),a(1),\dots,a(N)$ which determine the functional $A_N\xi$ and the Fourier coefficients $f_{\mu}(k)$ from the decomposition

$$\frac{\lambda^{2n}}{\left|1-e^{-i\lambda\mu}\right|^{2n}f(\lambda)}=\sum_{k=-\infty}^{\infty}f_{\mu}(k)e^{ik\lambda}.$$

The least favorable spectral density and the minimax (robust) spectral characteristic of the optimal linear estimation of $A_N\xi$ are found in the case where the spectral density $f(\lambda)$ is not known, but a class of admissible densities is given.

Department of Probability Theory, Statistics and Actuarial Mathematics, Kyiv National Taras Shevchenko University, Kyiv 01601, Ukraine
E-mail address: `maksim_luz@ukr.net, mmp@univ.kiev.ua`

EXTRAPOLATION OF PERIODICALLY CORRELATED WITH RESPECT TO TIME ISOTROPIC ON A SPHERE RANDOM FIELDS

I.I. Dubovetska, O.Yu. Masyutka, M.P. Moklyachuk

Let S_n be a unit sphere in the n-dimensional Euclidean space, let $m_n(dx)$ be the Lebesgue measure on S_n, and let $S_m^l(x),\ x\in S_n,\ m=0,1,\dots;\ l=1,\dots,h(m,n)$, be the orthonormal spherical harmonics of degree m. A continuous in mean-square random field $\zeta(t,x)$, $t\in\mathbb{Z}$, $x\in S_n$, is called periodically correlated (cyclostationary with period T) with respect to time isotropic on a sphere S_n if $\mathbb{E}\zeta(t+T,x)=\mathbb{E}\zeta(t,x)=0$, $\mathbb{E}\zeta(t+T,x)\zeta(s+T,y)=B\left(t,s,\cos\langle x,y\rangle\right)$, where $\cos\langle x,y\rangle=(x,y)$ is the “angular” distance between the points $x,y\in S_n$. This random field $\zeta(t,x)$ can be represented in the form

$$\zeta(t,x)=\sum_{m=0}^{\infty}\sum_{l=1}^{h(m,n)}\zeta_m^l(t)S_m^l(x),\quad \zeta_m^l(t)=\sum_{k=0}^{T-1}e^{2\pi ikt/T}\xi_{mk}^l(t)=\int_{S_n}\zeta(t,x)S_m^l(x)\,m_n(dx),$$

$\{\xi_{mk}^l(t)\}_{k=0}^{T-1}$, $m=0,1,\dots;l=1,\dots,h(m,n)$, are stationary stochastic processes.

We deal with the problem of the mean-square optimal linear estimation of the functional $A\zeta=\sum_0^{\infty}\int_{S_n}a(t,x)\zeta(t,x)\,m_n(dx)$ of the unknown values of a field $\zeta(t,x)$, $t\in\mathbb{Z}$, $x\in S_n$, from observations of the field $\zeta(t,x)$ for $t<0$, $x\in S_n$. Formulas are proposed for computing the value of the mean-square error and the spectral characteristic of the optimal linear estimate of the functional $A\zeta$. The least favorable spectral densities and the minimax (robust) spectral characteristics of the optimal estimates of the functional $A\zeta$ are determined for some special classes of spectral densities.

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Department of Probability Theory, Statistics and Actuarial Mathematics, Kyiv National Taras Shevchenko University, Kyiv, 01601, Ukraine
E-mail address: `mmp@univ.kiev.ua`

THE PROPERTIES OF THE STOPPING REGION IN LEVY MODEL

A. Moroz

We consider an optimal stopping problem in a Levy model, i.e. a model of finance market with single risk-free asset, whose price process is modeled by $\{X_t, t \in \mathbf{T}\}$, a homogeneous process with independent increments, finite moments and $X_0 = 0$. Here the parametric set $\mathbf{T}$ is either $[0, T]$ (finite time horizon) or $[0, \infty)$ (infinite time horizon). We find the conditions imposed on the payoff function g implying that the stopping region G is non-empty and conditions ensuring the region has a threshold structure.

For a finite time horizon, we prove the following results under some technical assumptions.

Theorem 1. *Optimal stopping region is nonempty for a pure jump Levy process with a finite symmetric Levy measure.*

Theorem 2. *Let one of the following assumptions hold: I) Jumps of process X_t are bounded from below; II) Levy measure of the process is symmetric. Then*

(1) *If g is such that $g''(x) \leqslant 0$ for all $x \in \mathbf{R}$ and $g''(x)$ is non-decreasing in x, then the stopping region G_t at any moment t has the following form: $G_t = [c(t), \infty)$.*

(2) *If g is such that $g''(x) \geqslant 0$ for all $x \in \mathbf{R}$ and $g''(x)$ is non-increasing in x, then G_t has the following form: $G_t = (-\infty, c(t)]$.*

For infinite time horizon, we have the following result.

Theorem 3. *Let A be the generator of the process X. Assume that there exists such x_1 that Ag is a non-zero positive measure on $(0, x_1)$ and a non-positive measure on (x_1, ∞) with*

$$\lim_{x\to\infty} Ag = l < 0, \ \mathbf{E}_x \left(\int_0^\infty e^{-qs} |Ag(X_s)| ds \right) < \infty.$$

Then the optimal stopping region has the structure $G = [x^*, \infty)$ with some $x^* \geqslant x_1$.

Kiev Taras Shevchenko University, Volodymyrska str. 64, 01601 Kyiv
E-mail address: mag-87@inbox.ru

ON OPTIMIZATION OF A PORTFOLIO OF INSURANCE CONTRACTS

B.V. Norkin

Insurance portfolio describes the structure of the aggregate premium obtained from different lines of insurance business.

Insurance portfolio optimization problem is set as a chance constrained stochastic optimization problem, where deduction from insurance reserves (net income, dividends) is maximized, subject to a constraint on the probability of ruin. We consider two approaches to solution of this problem.

In the first one [1], for a continuous time model when the risk process is a mixture (portfolio) of compound Poisson processes, the probability of ruin is replaced by its exponential (Cramer-Lundberg) upper bound. This trick allows one to eliminate a complicated probabilistic constraint and to decompose the problem according to separate lines of business. In this way, problems of optimization of insurance portfolios, tariffs, reinsurance treaties, and operational management are approximately solved. Results are illustrated by numerical examples.

In the second approach [2] for a discrete time model and in case of discrete distribution of random data the problem is reduced to a large scale mixed integer linear programming problem. The latter is solved by standard optimization tools. Numerical experiments were made for a test data set.

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Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, Glushkov avenue 40, Kiev, Ukraine
E-mail address: bogdan.norkin@gmail.com

SOME APPLICATIONS OF PROBABILISTIC METHODS OF OPTIMAL EXTRAPOLATION THEORY FOR THE NONLINEAR TASKS OF MODERN MARKET ECONOMY

A.D. Shatashvili[1], A.A. Fomin-Shatashvili, T.A. Fomina[2]

In this paper one general approach for computation of optimal extrapolation for some nonlinear evolutionary differential equation is proposed. The integration is produced with respect to a given Gaussian measure. Then the general results are applied, in particular, for increasing or decreasing forecast in time of a price $p(t)$ for certain product on market under the condition that the fixed price $p = p_0$ of supply $S(t, p(t))$ and demand $D(t, p(t))$ is given.

Let H be the real separable Hilbert space. Consider the system of nonlinear and linearized evolutionary differential equations of the form

$$\frac{dy(t)}{dt} - A(t)y(t) + A_1(t)y(t) + \alpha f(t, y(t)) = \xi(t) \quad (1)$$

and

$$\frac{dx(t)}{dt} - A(t)x(t) + A_1(t)x(t) = \xi(t) \quad (2)$$

$0 \le t \le a,\ y(0) = x(0) = \xi(0) = 0 \pmod P$, where α is a parameter of linear operators family $A(t)$ and $A_1(t)$, generally speaking unbounded;
$f(t, y(t))$- nonlinear function $\{[0, a] \times H\} \to H$,
$\xi(t)$-is a Gaussian process in $[0, a]$,
$M\xi(t) = 0$, with correlation function $R^2(t, s)$.

Under some sufficient conditions on coefficients of equation (1) the following results are proved.

1. Existence and uniqueness of solutions of equations (1) and (2), $y(t)$ and $x(t)$ correspondingly.
2. The measures μ_x and μ_y are equivalent ($\mu_x \sim \mu_y$), they are generated by solutions $y(t)$ and $x(t)$ on σ-algebra H, of bounded subsets of space H. Radon-Nikodym density $\frac{(d\mu_y)}{(d\mu_x)}(z)$ could be obtained in explicit form.
3. A method is constructed that uses Radon-Nikodym density $\frac{(d\mu_y)}{(d\mu_x)}(z)$ and gives a computation algorithm for optimal extrapolation of solution $y(t)$ at some point $t = T + h,\ h > 0,\ t, T + h \in [0, a]$, by observations of random process $y(t)$ up to time $t \le T$. Moreover, provided small nonlinearity of $\xi(t)$ under small α, this estimate expands the major term is a linear optimal forecast of Gaussian process into series by degrees of small parameter α.
4. Results received above are applied for Walris differential equation for product's price $p(t)$ in market.

$$\frac{dp(t)}{dt} = H[D(t, p(t), \alpha) - S(t, p(t))], \quad (3)$$

it simplifies to

$$\frac{dp(t)}{dt} - D(t, p_0)p(t) + S(t, p_0)p(t) + \alpha C(t, p(t))p_0 = 0, \quad (4)$$

with values p_0, $p(t)$, $D(t, p)$, $S(t, p)$ mentioned above, $C(t, p)$ be some known nonlinear function. If the right-hand side of equation (4) is disturbed by Gaussian random process $\xi(t)$, (stationary for simplicity), let it be Ornstein–Uhlenbeck process with $M\xi(t) = 0$ and correlation function $R^2_{\xi\xi} = M\xi(s)\xi(s + \tau) = e^{-|\tau|}$. Then it is easy to see that received equation coincides with (1) under some conditions, its averaged solution coincides with solution of (4).

Taking in account A.M.Yaglom's results concerning the problem of optimal forecast for stationary random processes, we get an explicit form of optimal forecast for Ornstein–Uhlenbeck Gaussian process. Then we can obtain an optimal estimate $\widehat{p}(t + h)$ in the explicit form for the forecast of price function $p(t)$ at the point $t = T + h$ using the average of solution of differential equation disturbed by Ornstein–Uhlenbeck process. If we expand this estimate with respect to a small parameter α, then the major term would be exactly the optimal forecast of Gaussian stationary process, mentioned above. It is well-known from A.M.Yaglom's results.

[1] Institute of applied mathematics and mechanics of NAS of Ukraine, Tkachenko st. 1/20, Donetsk
E-mail address: shatal@bk.ru
[2] Donetsk national university of economy and trade. Donetsk 83050, Schors str., 31.
E-mail address: fiva@inbo.ru

ASYMPTOTICS OF EXIT TIMES FOR DIFFUSION PROCESSES

Vladyslav Tomashyk

The problem of robustness of exit times for diffusion processes is quite important because of its application to the modeling of changing financial markets.

We consider the sequence of following stochastic differential equations

$$X_n(t) = X_n(0) + \int_0^t b_n(s, X_n(s))ds + \int_0^t \sigma_n(s, X_n(s))dW(s), n \ge 0 \quad (1)$$

with coefficients $b_n, \sigma_n : \mathbf{R}^+ \times \mathbf{R} \to \mathbf{R}$ that satisfy Yamada conditions (see [1], [2]). We assume the pointwise convergence of initial conditions and coefficients in (1):

$$X_n(0) \to X_0(0), b_n(t, x) \to b_0(t, x), \sigma_n(t, x) \to \sigma_0(t, x), n \to \infty. \quad (2)$$

Theorem 1. *Let $X_n(t)$ be the processes from (1) with Yamada conditions on coefficients. Under convergence (2) for $T > 0, \epsilon > 0 : P(\sup_{t\in[0,T]} |X_n(t) - X_0(t)| > \epsilon) \longrightarrow 0, n \to \infty$.*

Let $\tau_n, n \ge 0$ be the exit times of processes $X_n(t)$ above some level r where $r > X_n(0), n \ge 0$.

Theorem 2. *Let $X_n(t)$ be the processes from (1) with Yamada conditions on coefficients. Under convergence (2) for $\epsilon > 0$ the following convergence of exit times takes place: $P(|\tau_n - \tau| > \epsilon) \longrightarrow 0, n \to \infty$.*

The rate of convergence for exit times $\tau_n, n \ge 0$ is established.

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Kyiv National Taras Shevchenko University, Mechanics and Mathematics Faculty, Volodymyrska street, 64, Kyiv, Ukraine
E-mail address: vladdislav@gmail.com

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QUEUING THEORY

SPATIAL FILE-SHARING PROCESSES

F. Baccelli[1], F. Mathieu[2], I. Norros[3]

The following class of spatial models for peer-to-peer services is considered. Peers arrive to a metric space (D, d) (typically the Euclidean plane or a torus) according to a Poisson process (typically spatially uniform w.r.t. Haar measure). Peers at points x and y provide service (e.g., upload chunks of a file) to each other in both directions at rate $f(d(x, y))$, where f is non-increasing. Each peer has an independent exponentially distributed service demand, and it leaves the system as soon as the cumulative service it has received reaches its demand. This new type of model turns out to have very interesting properties. The system is readily seen to be super-scalable, that is, the higher the input rate, the lower the mean service time. Second, when f is strictly decreasing, the system has a repulsive character, since peers вЂњtry to kill each otherвЂќ more strongly when they are close. We prove this kind of behaviour in the torus case using Palm calculus. The system's overall performance turns out to depend on a single basic parameter. The extreme cases are a densely populated system with very fast service and a system whose stationary spatial distribution resembles Matern's hard core point process and that has significantly poorer performance. We present accurate approximate formulae describing the performance continuum between the extreme regimes. Finally, we discuss problems of existence and uniqueness of the process on the infinite plane. These turn out to be challenging because of non-monotonicity features related to the repulsivity mentioned above.

[1] INRIA, France
E-mail address: `Francois.Baccelli@ens.fr`
[2] INRIA, France
E-mail address: `Fabien.Mathieu@inria.fr`
[3] VTT Technical Research Centre of Finland
E-mail address: `ilkka.norros@vtt.fi`

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FAST SIMULATION OF THE PROBABILITY OF TWO MARKOV CHAINS INTERSECTION

O.M. Khomyak

The behavior of a repairable system in the regeneration period is described by a Markov chain $\{\xi_n,\ n \geq 0\}$ with a finite set of states U and transition probabilities $\{p_{\nu\mu},\ \nu, \mu \in U\}$. We suppose that the state $\theta \in U$ corresponds to the state when all components are in working states, $\xi_0 = \theta$ with probability 1. In order to distinguish transitions associated with failures we introduce an indicators $\{a_{\nu\mu}\}$: $a_{\nu\mu} = 1$ if transition $\nu \to \mu$ is connected with a failure of some component, and $a_{\nu\mu} = 0$ otherwise. With each state $\nu \in U$ one can associate an efficiency of the system defined by the function $f(\nu)$ (available efficiency). At the same time the required efficiency of the system is defined by the state of another Markov chain $\{\eta_n, n \geq 0\}$ with a finite set of states $I \in \{0, 1, 2, \ldots\}$ and transition probabilities $\{q_{ij},\ i, j \in I\}$. With each state $j \in I$ one can associate a required efficiency $g(j)$. If the required efficiency becomes higher than the available efficiency, then system fails (it is so called functional failure). Denote by $\tau = \min\{n : \xi_n = \theta\}$ the duration of the regeneration period. The main aim of the investigation was to evaluate the probability $q = \mathbf{P}\{\tau_F < \tau\}$ of system's failure in a regeneration period where $\tau_F = \min\{n : g(\eta_n) > f(\xi_n)\}$.

In order to evaluate the probability q we propose a fast simulation method being a generalization of corresponding method in [1]. It is proved that under some weak conditions the relative error of estimates remains bounded when the reliability of components increases, and moreover this relative error even decreases. One of the advantages of the method proposed is the simplicity of realization on a personal computer.

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Cybernetics Institute, National Academy of Sciences of Ukraine, Kiev
E-mail address: `khomyak.olya@mail.ru`

ASYMPTOTIC ANALYSIS OF QUEUEING SYSTEM WITH TWO INPUT FLOWS

I.N. Kuznetsov

A queueing system with two input flows of customers is considered. This system can be used as a model describing the behavior of a cell with fully accessible service of customers of the first type and with a queue for customers of the second type [1].

A system consists of two sets of service devices containing m and n devices respectively. There are two Poisson input flows of h-customers with the rate λ_1 and o-customers with the rate λ_2. The service device for arriving h-customer is taken from the first set (if there is any free). If all devices of the first set are occupied then h-customer takes any free device of the second set. If both sets have no free devices then h-customer leaves the system unserved (system failure of the first type). Arriving o-customers can be served only by devices of the first set. If all m devices are occupied then o-customer takes one of free places (if any) in a queue. The total number of places in a queue equals r. If all places in a queue are occupied then o-customer leaves the system unserved (system failure of the second kind). Service times on first and second sets of service devices are independent exponentially distributed random variables.

We investigate the probability $q^{(i)}$ of system failure of kind i in a busy period (at least one customer is present in the system). System failure is called monotone if no customers were served in a busy period until the system failure occurs. Necessary and sufficient conditions when the probability of system failure in a busy period is equivalent to the probability of monotone failure (for both types of failures) are obtained.

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V.M. Glushkov Institute of Cybernetics of the National Academy of Sciences, 40, Prospect Akademika Glushkova, 03680 Kyiv, Ukraine

E-mail address: sea_hawk@inbox.ru

FAST SIMULATION OF THE NONSTATIONARY FAILURE RATE OF THE REPAIRABLE SYSTEM

N.Yu. Kuznetsov[1], A.A. Shumska[2]

Consider a system consisting of m components. Each component can be one of three following types.

C_1: Repairable components with immediately detected failures (revealed failures); the component of this type is characterized by distribution functions $F_i(x)$ and $G_i(x)$ of failure-free operation and repair time respectively.

C_2: Periodically tested components (failure remains unrevealed until the next test; then repair starts immediately); in addition to distribution functions $F_i(x)$ and $G_i(x)$ of failure-free operation and repair time, the component of this type is characterized by the constant test interval A_i, and by the time to the first test $A_i^{(0)}$. The time of the test is supposed to be negligible small.

C_3: Unrepairable components (a component remains in a failed state to the end of system operation time); the component of this type is characterized by the distribution function $F_i(x)$ of failure-free operation time only.

The system reliability structure is characterized by minimal failure cut sets. The system operates in a fixed time interval $(0,\ T)$. We propose a fast simulation method generalized compared with [1] which makes it possible to evaluate the failure rate of the system simultaneously in the set of points $0 < t_1 < t_2 < ... < t_n \leq T$.

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[1] V.M. Glushkov Institute of Cybernetics of the National Academy of Sciences, 40, Prospect Akademika Glushkova, Kyiv 03680, Ukraine

E-mail address: sea_hawk@inbox.ru

[2] National Technical University of Ukraine "Kyiv Polytechnic Institute", 37, Prospect Peremogy, Kyiv 03056, Ukraine

E-mail address: shumska-aa@ukr.net

NUMERICAL INVESTIGATION OF THE RQ-SYSTEM WITH CONFLICTS OF REQUESTS

A.A. Nazarov, E.A. Sudyko

In this paper we consider numerical methods of investigation of RQ-systems with conflicts of requests.

Such systems have random input flow of requests. Each request tries to capture the single server for servicing, and if the server is free at this moment, it occupies it for a service time. If the server is busy, then the arriving request and request under service collide and are sent to a retrial pool. The requests stay at the retrial pool for a delay time, which is exponentially distributed. After a random delay, each request from the retrial pool makes a retrial attempt to capture the server. And if the server is free, it occupies it for a service. Another side, request under service and retrial attempt request are sent to a retrial pool for a random delay.

We have proposed complex of numerical methods, which allow us with high precision to find characteristics of considering systems under condition of knowing values of parameters for prelimit situation.

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National Research Tomsk State University

E-mail address: esudyko@yandex.ru

ON A CLASS OF QUEUEING SYSTEMS WITH SERVER'S VACATION AFTER BUSY PERIOD COMPLETION

M.Ya. Postan[1] and N.V. Rumyantsev[2]

The queueing systems are considered in which server is switched off for a random period of time (vacation period) after busy period completion. During the server's vacation the new customers may arrive but with another rate. Vacation period is depended on number of customers served during the last busy period. The two types of such queueing systems are studied in details: a) Markovian systems which behavior is described by generalized "birth and death" process; b) systems of M/G/1 type with feedback. In equilibrium, for both types of systems the state-probabilities are calculated and corresponding ergodicity conditions are found.

[1] Odessa National Maritime University, Department "Economics & Management", Mechnikov Str. 34, 65029, Odessa, Ukraine
E-mail address: postan@ukr.net

[2] Donetsk National Technical University, Department "Computer Sciences and Technologies", Artem Str. 58, 83001, Donetsk, Ukraine
E-mail address: mmme@dongu.donetsk.ua

A JACKSON NETWORK IN THE NON-DEGENERATE SLOWDOWN REGIME

Yair Y. Shaki

We consider a Jackson network in the non-degenerate slowdown regime, that is, a heavy traffic diffusion regime in which delay and service times are of the same order. We also assume that each customer may abandon the system while waiting. We show that in this regime the queue-length process converges to a multi-dimensional regulated Ornstein-Uhlenbeck process. Finally, we represent the law of the delay and service time processes.

Department of Electrical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel
E-mail address: y_shaki@hotmail.com

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SYSTEM WITH PARALLEL SERVICE OF THE MULTIPLE REQUESTS AS A MATHEMATICAL MODEL OF THE FUNCTIONING OF AN INSURANCE COMPANY

I.A. Sinyakova, S.P. Moiseeva

In insurance company values of insurance premiums and possible indemnity are strictly specified for every insurance contract. They are unknown until the conclusions of the contract and should be considered as a random values. Moments of receipt of insurance premiums and occurrence of insurance claims are also random values [1]. Therefore, it is actual to investigate more complex models of insurance companies, which allow to take in consideration changes of capital and effect of number of risks on it. Consider system with parallel service of requests of mixed types as a mathematical model of the functioning of an insurance company [2]. An insurance company enters into contracts of two types (for example, life insurance, property insurance). It allows for the conclusion of contracts of first and second types at the same time. Service discipline for this systems assumes that first type contracts comes in the first block of service, second type contract comes in the second block and occupies any of the empty servers for service. Service times for both types of contracts are exponentially distributed with parameters μ_1 and μ_2. The key indicators of the effective functioning of the insurance company are number of risks and capital of company. Therefore, three-dimensional random process $\{k_1(t), k_2(t), S(t)\}$ defines state of the insurance company at the moment t, where $k_{1,2}(t)$ is number of risks for contracts of first and second types respectively, $S(t)$ is capital of the company at the moment t. As a result of investigation we have found formula for definition mean value and dispersion of the changes in capital of insurance company.

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National Research Tomsk State University, Tomsk
E-mail address: irinka_asf@mail.ru, smoiseeva@mail.ru

ON RETRIAL QUEUES CONTROLLED BY THE THRESHOLD AND HYSTERESIS SRATEGIES

I. Usar, E. Lebedev, I. Makushenko

We deal with a m-server retrial queues of the type $M_Q/M/m/\infty$ in which a rate of primary call flow $\lambda^{(j)}$ depends on the number j - of customers in the orbit. The rates of repeated calls ν and of the service process μ are supposed to be constant. The effective approach to find a steady state distribution of the service process is proposed. It consists of two stages. At the first stage we construct the explicit vector-matrix form of the stationary distribution for the queue with finite orbit. At the second stage the stationary probabilities for the queue with infinite orbit are approximated by the formulae got at the first stage.

Variable rate of the input flow allows to consider the queue controlled by different strategies. In the report we consider two strategies: threshold and hysteresis strategies. Threshold strategy realizes the following algorithm of the service process control: we set $\lambda^{(j)} = \lambda^{(1)}$ if $j = 0, 1, ..., h$ and $\lambda^{(j)} = \lambda^{(2)}$ if $j = h + 1, ...$

Hysteresis strategy may be introduced by means of the two thresholds $0 < h_1 \leqslant h_2$. The rate of input flow changes from $\lambda^{(1)}$ to $\lambda^{(2)}$ if the number of repeated calls reaches the level h_2 from below and it changes from $\lambda^{(2)}$ to $\lambda^{(1)}$ if the number reaches h_1 from above. If the number of repeated calls is in the interval $[h_1, h_2)$ than the queue follows the mode in which it was at previous moment of time. It is clear that class of threshold strategies is the subset of the class of hysteresis strategies . The similar controlled systems with single-server have been considered in [1],[2]. An analysis of the multiserver models is contained in [3].

The explicit formulae for steady state probabilities enable to propose an effective solving algorithm for optimal decision making which consists in finding optimal positions for the levels to maximize some objective function. As such a function we took the following one $W = C_1S_1 - C_2S_2 - C_3S_3$, where S_1 is the number of calls have being served; S_2– the number of calls that become repeated calls; S_3– the number of switching of the input flow; C_1 – profit associated with service of a call; C_2 – penalty for refusal to serve; C_3 – penalty for switching of the input flow rate.

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Kyiv Taras Shevchenko National University, Cybernetics, av.Glushkowa 4-D, Kiev, Ukraine
E-mail address: usar@unicyb.kiev.ua

STABILITY BOUNDS FOR SOME QUEUEING MODELS

A.I. Zeifman, A.V. Korotysheva, Y.A. Satin, G.N. Shilova

In the early 1970s, B. V. Gnedenko et al performed the first investigations of nonstationary birth-death queueing models, see [1, 2]. In the papers [5, 6, 7] we give the general approach to the study of nonhomogeneous birth-death processes, and obtain accurate estimates of their speed of convergence and stability.

Here we deal with a new class of Markovian queues with batch arrival and group services, see the first results in [3, 4].

Namely, we consider the Markovian queue with the non-zero transition rates $q_{k,k+n} = \lambda_n(t)$ and $q_{k,k-n} = \mu_n(t)$ for arrival and service of the group of n customers respectively.

We obtain the stability bounds for the queue-length process of this model and consider some concrete simple examples.

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Vologda State Pedagogical University, S.Orlova, 6, Vologda, Russia
E-mail address: a_zeifman@mail.ru

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STOCHASTIC MODEL FOR COMMUNICATING RETRIAL QUEUEING SYSTEMS WITH CYCLIC CONTROL IN RANDOM ENVIRONMENT

A.V. Zorine

Two communicating queueing systems are considered. Input flows are formed in an external environment with a finite number of states. Depending on the environment's state each input flow is a non-ordinary Poisson flow with parameters determined by the environment. Each queueing system has two conflicting [1] input flows. Service durations in each system are random, stochastically dependent with different probability distributions. Several customers can be serviced during one servicing period. Saturation flows [1] are used to define service process. A cyclic algorithm with fixed durations is used in each queueing system to control input flows. To resolve conflictness, a readjustment period is inserted after each service period. After servicing in the first queueing system customers from the first input flow are redirected to the second queueing system, transition requiring random time for completion as described in [1]. Customers from the second flow in the first queueing system independently from each other may also be redirected to the second queueing system. So one of the input flows entering the second queueing system is a flow of retrial customers. Using the cybernetic approach a mathematical model for the communicating queueing systems is constructed in form of a multidimensional discrete denumerable Markov chain. Classification of states of this Markov chain is carried out.

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N.I. Lobachevsky State University of Nizhni Novgorod, Department of computational mathematics and cybernetics, 603950, pr. Gagarina, 23, bld. 2, Nizhni Novgorod, Russia
E-mail address: zoav1602@gmail.com

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ON SPECTRAL PROPERTIES OF LARGE SYMMETRIC DILUTE RANDOM MATRICES

O. Khorunzhiy

We consider an ensemble of $n \times n$ real symmetric matrices $H^{(n,\rho)}$ of the form

$$H_{ij}^{(n,\rho)} = a_{ij}\, b_{ij}^{(n,\rho)}, \quad 1 \leqslant i \leqslant j \leqslant n, \tag{1}$$

where $\{a_{ij}, 1 \leqslant i \leqslant j\}$ and $\{b_{ij}^{(n,\rho)}, 1 \leqslant i \leqslant j \leqslant n\}$ are families of jointly independent random variables such that a_{ij} are of the same probability law that is symmetric and has several first moments finite, $V_k = \mathbf{E}|a_{ij}|^k < \infty$; random variables $b_{ij}^{(n,\rho)}$ are proportional to the Bernoulli ones, $b_{ij}^{(n,\rho)} = 1/\sqrt{\rho}$ with probability ρ/n and $b_{ij}^{(n,\rho)} = 0$ with probability $1 - \rho/n$, $0 < \rho \leqslant n$. The ensemble (1) with $\rho = n$ is widely known as the Wigner ensemble of random matrices. We are interested in the asymptotic behavior of the spectral norm $\|H^{(n,\rho)}\| = \lambda_{\max}^{(n,\rho)}$ in the limit of infinite n and ρ.

Theorem 1. *Let the probability distribution of a_{ij} be such that for some $\phi > 0$ the moment $V_{12+2\phi}$ exists. Then for any sequence $\rho_n = n^{2/3(1+\varepsilon)}$ with given $\varepsilon > 3/(6+\phi)$, the limiting probability*

$$\limsup_{n\to\infty} \mathbf{P}\left(\lambda_{\max}^{(n,\rho_n)} \geqslant 2\sqrt{V_2}\left(1 + \frac{x}{n^{2/3}}\right)\right) \leqslant G(x), \quad x > 0 \tag{2}$$

admits a universal upper bound in the sense that $G(x)$ does not depend on the values of V_{2l}, $2 \leqslant l \leqslant 6$ and $V_{12+2\phi}$; in particular, relation (2) is true with $G(x) = e^{-Cx^{3/2}}$.

The proof is based on the study of high moments $L_n = \mathbf{E}\sum_{i=1}^n (\hat{H}_n^{2s_n})_{ii}$, $s_n = \chi n^{2/3}$, $\chi > 0$ of matrices $\hat{H}_n = \hat{H}^{(n,\rho_n)}$ of the form (1) with truncated random variables $\hat{a}_{ij}$. We study L_n with the help of a generalization [1] of the approach developed in [2, 3].

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University of Versailles - Saint-Quentin, 78035 Versailles, France
E-mail address: oleksiy.khorunzhiy@uvsq.fr

ON LIMITING LAWS OF FLUCTUATIONS FOR THE CERTAIN SPECTRAL STATISTICS OF THE WIGNER MATRICES

A. Lytova

We continue investigations started in [1, 2, 3] where the limiting laws of fluctuations were found for linear eigenvalue statistics $\mathrm{Tr}\varphi(M^{(n)})$ and for the normalised matrix elements $\sqrt{n}\varphi_{jj}(M^{(n)})$ of differentiable functions of real symmetric Wigner matrices $M^{(n)}$ as $n \to \infty$. Here we consider another spectral characteristic of Wigner matrices, $\xi_n^A[\varphi] = \mathrm{Tr}\varphi(M^{(n)})A^{(n)}$, where $\{A^{(n)}\}_{n=1}^{\infty}$ is a certain sequence of non-random matrices. We show first that if $M^{(n)}$ belongs to the Gaussian Orthogonal Ensemble, then $\xi_n^A[\varphi]$ satisfies the Central Limit Theorem. Then we consider Wigner matrices with i.i.d. entries possessing entire characteristic function and find the limiting probability law for $\xi_n^A[\varphi]$, which in general is not Gaussian.

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B.Verkin Institute for Low Temperature Physics and Engineering, 47, Lenin Ave., Kharkov 61103, UKRAINE
E-mail address: anna.lytova@gmail.com

LIMITING LAWS FOR OF SPECTRAL STATISTICS OF LARGE RANDOM MATRICES

L. Pastur

We consider certain functions of eigenvalues and eigenvectors (spectral statistics) of real symmetric and hermitian random matrices of large size. We first explain that an analog of the Law of Large Numbers is valid for these functions as the size of matrices tends to infinity. We then discuss the scale and the form for limiting fluctuation laws of the statistics and show that the laws can the standard Gaussian (i.e., analogous to usual Central Limit Theorem for appropriately normalized sums of independent or weakly dependent random variables) in non-standard asymptotic settings, certain non Gaussian in seemingly standard asymptotic settings, and other non Gaussian in non-standard asymptotic settings.

Institute for Low Temperature Physics, Kharkiv, Ukraine
E-mail address: lpastur@ilt.kharkov.ua

CENTRAL LIMIT THEOREM FOR LINEAR EIGENVALUE STATISTICS OF β MATRIX MODELS AND ITS APPLICATIONS.

M.V. Shcherbina

We consider β-matrix models with real analytic potentials for both one-cut and multi-cut regimes. The recent results on the asymptotic behavior of the characteristic functional of linear eigenvalue statistics, in particular, non gaussian behavior of the characteristic functional in the multi-cut regime will be discussed. The applications of these results to the proof of the universality conjecture for β-matrix models will be also considered.

Institute for Low Temperature Physics Ukr.Ac.Sci, 47 Lenin ave, Kharkov, Ukraine
E-mail address: shcherbi@ilt.kharkov.ua

ON THE RANDOM MATRIX ENSEMBLES RELATED TO THE CLASSICAL COMPACT GROUPS

V. Vasilchuk

We deal with the random matrix ensembles whose probability distributions are invariant under transformations from the classical compact groups (unitary, orthogonal and symplectic). We are interested in the asymptotic behavior of the eigenvalues, as the size of matrices tends to infinity, and prove the analogues of the law of large numbers and the central limit theorem for the linear eigenvalue statistics with smooth test functions. The connections with the non-commutative free probability studies are also considered.

B.Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Ave., Kharkov 61103, UKRAINE
E-mail address: vasilchuk@ilt.kharkov.ua

STRUCTURE OF SUBCLASSES IN THE FRAME OF HEAVY-TAILED DISTRIBUTIONS

Anastasios G. Bardoutsos, Dimitrios G. Konstantinides

The structure of the heavy tails has been studied extensively. The most known result is given by figure [4, Fig. 1.4.1], which describes the relationship between the main classes of distributions such as long tails, subexponentials tails , dominatedly varying tails, regular varying tails and slowly varying tails. As it custom we denote them as $\mathcal{L}$, $\mathcal{S}$, $\mathcal{D}$, $\mathcal{R}_{-a}$, $a > 0$ and $\mathcal{R}_0$. It's common truth that more classes have been introduce to support the demands in different areas of applied probability such as risk theory. As a result plenty counterexamples have been given to sketch the complicate structure.

On the other hand the distribution of main interest and not the designed examples, can be presented in a more simple relation. Roughly speaking, the subexponential class can be divided into two disjoint parts as

$$\mathcal{S} \approx (\mathcal{D} \cap \mathcal{L}) \cup (\mathcal{S} \cap \mathcal{R}_{-\infty}), \tag{1}$$

where $\mathcal{R}_{-\infty}$ is the class of rapidly varying tails (see Definition [4, A3.11]).

We will present two commonly fulfilled assumptions. All the distributions that fulfill this assumptions not only satisfy relation (1) as equality, but also the structure of classes of the extended rapidly varying, slowly varying and constantly varying tails (see Definitions[1,], [4, A3.1] and [3, p.118]) is given in an also simple relation. The results are based on the relation between hazard rates functions and distributions,which is a well known fact see for instance [5], and the aid of Matuszewska indices (see [2]).

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Department of Statistics and Actuarial Science,University of the Aegean, Karlovassi, GR-83 200 Samos, Greece
E-mail address: sasm10007@sas.aegean.gr, konstant@aegean.gr

GALERKIN APPROXIMATION WITH LAGUERRE FUNCTIONS FOR EQUATION OF RISK THEORY

V.A. Chernecky

Consider the ordinary renewal model of functioning of an insurance company:
(a) *The claim sizes* $(Z_n)_{n\in\mathbb{N}}$ *are positive independent identically distributed (iid) random variables (rvs)* $(Z_n = Z)$ *having the distribution* $F(v)$, $F(0) = 0$, *characteristic function* $\Phi_Z(\xi)$ *and finite mean* $\mu = \mathsf{E}Z$;
(b) *The inter-arrival times* $(T_n)_{n\in\mathbb{N}}$ *are positive iid rvs* $(T_n = T)$ *having the distribution* $K(v)$, $K(0) = 0$, *characteristic function* $\Phi_T(\xi)$ *and finite mean* $\mathsf{E}T = 1/\alpha$;
(c) $c > \alpha\mu$ *is the gross premium rate*;
(d) *The sequences* $(Z_n)_{n\in\mathbb{N}}$ *and* $(T_n)_{n\in\mathbb{N}}$ *are independent of each other.*
(e) *At last one of the variables* Z *or* T *has a distribution density.*

Ruin-probability, $\psi(u)$, of the insurance company with initial capital $u \geqslant 0$, in ordinary renewal process, satisfies the fundamental equation of risk theory [2, 3, 5]:

$$(A\psi)(u) \equiv \psi(u) - \int_0^\infty dK(v) \int_0^{u+cv} \psi(u + cv - w)\, dF(w) = 1 - \int_0^\infty F(u + cv)\, dK(v), \tag{1}$$

with the boundary condition

$$\lim_{u\to+\infty} \psi(u) = 0. \tag{2}$$

Using the Wiener-Hopf method [4, 7], the solvability theory for the problem (1)-(2) is constructed. The equation (1) turns out to be one-sided Wiener-Hopf equation of nonnormal type since the symbol $\mathcal{A}(\xi)$ of the operator A in (1), given by the formula

$$\mathcal{A}(\xi) = 1 - \Phi_T(-c\xi)\, \Phi_Z(\xi), \quad \xi \in \overline{\mathbb{R}}, \quad \mathcal{A}(\infty) = 1, \tag{3}$$

has an unique zero-point at $\xi = 0$ and this zero-point is simple.

Applying the results of A. Pomp [6], sufficient conditions are obtained for the convergence of the Galerkin process with system of Laguerre functions in L^p-norm for the problem (1)-(2).

The following process to the approximate solution of the equation

$$(A\psi)(u) = f(u) \tag{4}$$

is called the Galerkin process with the system of Laguerre functions $\{\mathcal{L}_n(u)\}_{n=0}^{\infty}$:

An approximate solution $\psi_n(u)$ *of the equation* (4) *is sought in the form of a linear combination*

$$\psi_n(u) = \sum_{k=0}^{n} \xi_k^{(n)} \mathcal{L}_k(u).$$

The coefficients $\xi_k^{(n)}$ *are determined from the equation system:*

$$\sum_{k=0}^{n} \xi_k^{(n)} \langle A\mathcal{L}_k, \mathcal{L}_j\rangle = \langle f, \mathcal{L}_j\rangle, \quad j = 0, 1, \dots, n. \tag{5}$$

Following A. Pomp [6], introduce the integer number $\varkappa$ and real number ω depending on p by the following table (in the notation of [6], $M = 1$):

1.	$1 \leqslant p \leqslant \frac{4}{3}$	$\varkappa = 3$	$\omega = 0$
2.	$\frac{4}{3} < p < 2$	$\varkappa = 2$	$\omega = 0$
3.	$p = 2$	$\varkappa = 1$	$\omega = 0$
4.	$2 < p$	$\varkappa = 2$	$\omega > \frac{1}{2} - \frac{1}{p}$

We arrive at the following result about the applicability of the Galerkin process to the problem (1)-(2).

Theorem 1. *If for the right side*

$$f(u) = 1 - \int_0^{\infty} F(u + cv)\, dK(v)$$

of the equation (1) *the following three conditions are fulfilled:*

a) $f(u) \in AC_{\text{loc}}^{2\varkappa-1}$ *(besides* $u = 0$*),*

b) all elements of sequence $\{\langle f, \mathcal{L}_n\rangle\}_{n=0}^{\infty}$ *exists,*

c) $|u + i|^{\omega} \frac{d^j}{du^j}[u^{\varkappa} f(u)] \in L^p, \quad j = 0, 1, \dots, 2\varkappa,$

then the Galerkin process is applicable to the problem (1)-(2) *in the space* L^p*.*

The writing $f(u) \in AC_{\text{loc}}^{h}$ means that for $h \in \mathbb{N}$ there exists a function $f_0(u)$ almost everywhere coinciding with $f(u)$, having absolutely continuous derivatives until the order h on any finite interval and $(h+1)$-differentiable [1].

Note that the right side of the equation (1) is the tail of the random variable $Z - cT$, which plays decisive role in the theory.

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Odessa State Academy of Refrigeration, Department of Higher Mathematics, ul. Dvoryanskaya 1/3, 65026, Odessa, Ukraine

E-mail address: chern.va@paco.net

CHARACTERIZATION THEOREMS FOR CUSTOMER EQUIVALENT UTILITY INSURANCE PREMIUM CALCULATION PRINCIPLE

Vitaliy Drozdenko

Characterization theorems for several properties possessed by customer equivalent utility insurance premium calculation principle are presented. Demonstrated theorems cover cases of additivity, consistency, iterativity, and scale invariance properties. Results are formulated in a form of necessary and sufficient conditions for attainment of the properties imposed on the customer utility function. Characterizations for customer zero utility principle are formulated as corollaries of corresponding characterizations for customer equivalent utility principles. We show also that for customer zero utility principle subjected to pricing of only strictly positive risks, class of utility functions producing scale invariant premiums is larger than in the general case.

Dragomanov National Pedagogical University, Pyrogov St. 9, Kiev, Ukraine

E-mail address: drozdenko@yandex.ru

EQUILIBRIUM'S CONDITION BETWEEN THE RUIN AND SURVIVING

Gusak (Husak) Dmytro

Let $\xi_t = P_t - S_t$ $(t \geq 0, \xi_0 = 0)$ be the risk process with the initial capital $u = 0$ and with the claim's process

$$S_t = \sum_{k \leq N_1(t)} Y_k$$

$(0 > Y_k$ – arbitrary distributed) and premium's processes:

1) $P_t = Ct,\ 0 < C$ – a gross premium rate;

2) $P_t = \sum_{k \leq N_2(t)} X_k$ $(0 < X_k$ – exponentially distributed premiums); $N_{1,2}(t)$ – Poisson processes with intensities $\lambda_{1,2} > 0$. The safety security loading is denoted δ(sometimes ρ)

$$\delta = \rho = \frac{E\xi_1}{ES_1} > 0, ES_1 = \lambda_1\mu, \mu = EY_1 > 0.$$

Let us denote estimates of compensation premiums functions (see ch.4 [1])

a) $P_{(a)} = ES + \alpha_0 ES, S = S_1, \alpha_0 > 0;$

b) $P_{(b)} = ES + \alpha\sigma(S),\ \sigma(S) = \sqrt{DS}, \alpha > 0;$

c) $P_{(c)} = ES + \beta\sigma^2(S), \beta > 0;$

α_0, α, β – level's parameters, where estimates

a) corresponds to the expected value primiple,

b) – to the standard deviation principle,

c) – to the variance principle.

In terms of the ruin and surviving probabilities

$$q_+ = \mathbf{P}\{\inf_{0 \leq t < \infty} \zeta_t < 0\} = \Psi(0),\ p_+ = 1 - q_+$$

it is established (see [2]): 1) $\delta = \alpha_0 = \frac{p_+}{q_+}$, 2) $\delta = \frac{p_+}{q_+ - p}$, $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. Under the balance condition $(p_+ = q_+)$

1) $\delta_* = \alpha_0^* = 1$; 2) $\delta_* = \frac{1}{1-2p} = \frac{\lambda_1+\lambda_2}{\lambda_2-\lambda_1} > 0$, iff $\lambda_2 > \lambda_1$, and premium's estimates are defined.

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Institute of Mathematics of National Academy of Sciences of Ukraine, Tereshchenkivska 3, Kyiv, Ukrain

E-mail address: gusak_dv@mail.ru

QUASI-STATIONARY DISTRIBUTIONS FOR PERTURBED REGENERATIVE PROCESSES WITH DISCRETE TIME

Mikael Petersson

Nonlinearly perturbed discrete time regenerative processes with regenerative stopping times are considered. We define the quasi-stationary distributions for such processes and present conditions for their convergence. Under some additional conditions, the quasi-stationary distributions can be expanded in an asymptotic power series with respect to the perturbation parameter. We give an explicit recurrence algorithm for calculating the coefficients of this series. Applications to discrete time risk processes and birth-death type processes with absorption are presented.

Stockholms universitet, Matematiska institutionen, SE-106 91, Stockholm, SWEDEN

E-mail address: mikpe@math.su.se

FUNCTIONAL PROPERTIES OF THE SURVIVAL PROBABILITIES IN SOME RISK MODELS AND THEIR APPLICATIONS

E.Yu. Ragulina

We consider the classical risk model and the risk model with stochastic premiums [1] when an insurance company invests its surplus to the financial (B,S)-market and a price of risk asset follows a jump process $S(t) = S(0)e^{r_{st}t + R_{st}(t)}$, where r_{st} is a positive constant, $S(0) > 0$ is a price of the risk asset when $t = 0$, $R_{st}(t) = \sum_{i=1}^{N_{st}(t)} Y_i^{st}$, $N_{st}(t)$ is a homogeneous Poisson process with parameter $\lambda_{st} > 0$ $(N_{st}(0) = 0)$, Y_i^{st} $(i = \overline{1, \infty})$ are i.i.d. r.v.'s with d.f. $F_{st}(y)$ $(0 < F_{st}(0) < 1)$, finite exponential moments and $\mathbf{M}Y_i^{st} = 0$. Upper and lower estimates are found for the finite-horizon survival probabilities in these risk models. Continuity and differentiability properties of the finite-horizon and infinite-horizon survival probabilities are investigated, derivatives of these functions with respect to the initial surplus are estimated. These estimates are used for derivation of relations connecting accuracy and reliability of the survival probability approximations with their statistical estimates. Continuity and differentiability properties of the survival probabilities are investigated in [2] when an insurance company invests its surplus to risk-free asset only, and in [3] these properties are used for finding analytical expressions for the survival probabilities.

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Donetsk National University, Department of Probability Theory and Mathematical Statistics, Universitetskaya 24, Donetsk, Ukraine

E-mail address: lena_ragulina@mail.ru

DIVIDENDS AND CAPITAL INJECTIONS IN NON-LIFE INSURANCE

H. Schmidli

The classical measure for risk in non-life insurance is the ruin probability. The problem is that this measure does not depend on the time to ruin nor on the deficit at ruin. De Finetti [2] introduced the expected discounted dividend payments. This measure, however, does not take into account the needs of the policy holders. We therefore consider two similar measures: The expected discounted capital injections and the expected discounted capital injections minus dividends. That is, whenever the surplus becomes negative, capital has to be injected in order to make the surplus positive. We show how these risk measures can be calculated and how optimal reinsurance treaties can be found. The material presented is joint work with Natalie Scheer and Julia Eisenberg.

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Institute of Mathematics, University of Cologne, Weyertal 86-90, 50931, Cologne, Germany

E-mail address: schmidli@math.uni-koeln.de

APPROXIMATION METHODS FOR AMERICAN TYPE OPTIONS

Dmitrii Silvestrov

Lecture presents a survey of results from a new book on approximation methods for optimal pricing for American type options. At the moment, the book is at the final stage of preparation. The book contains a detailed presentation of approximation methods for rewards of American type options for multivariate modulated Markov price processes with discrete time. The classes of price processes under consideration include modulated multivariate Markov chains, modulated random walks, and various autoregressive type models. General convergence results are presented, as well as their applications to space skeleton approximations, tree approximations, and Monte Carlo based approximation algorithms for option rewards. Also, results related to studies of structure for optimal stopping domains are presented as well as results related to option reselling problem. Theoretical results are illustrated by results of experimental studies. Finally, connection with problems of optimal stopping and convergence for modulated multivariate Markov type price processes with continuous time is discussed.

Stockholm University

E-mail address: silvestrov@math.su.se

SYSTEM ANALYSIS OF RISK PROCESSES

M.R. Zvizlo

The research of work of the insurance companies requires all new tools in order to avoid and minimize the risk of bankruptcy. During the study of work of the insurance company, the process of the risk which describes its work, is considered. When changing the insurance conditions, the process of risk is also modified because there is a problem in the studying other than classical risk processes . Consider the next model of financial risk of the insurance company, let the process of risk be given as follows

$$R_t(u) = u - U_t + A_t,$$

where u is the initial capital of the insurance company at the moment $t = 0$; U_t, A_t - are reduction processes.

Let's mark $\tau = \inf\{t > 0 : R_t(u) < 0\}$ a moment of bankruptcy of insurance. Let's assume $\tau = +\infty$, if $R_t \geq 0 \forall t > 0$. The transformations are conducted, which allow to get the solution of risk for the probability of bankruptcy of the insurance company, the assessments of the bankruptcy and the amount of loss assessment of the insurance company are found, which allow to optimize the work of the insurance companies under conditions of uncertainty.

Ivan Franko National University of L'viv, Department of Theoretical and Applied Statistics, Lviv, Universytetska, 1, 79000

E-mail address: zvmax@i.ua

CONSISTENT CRITERIA OF CHECKING HYPOTHESES IN HILBERT SPACE OF MEASURES

Lela Aleksidze, Zurab Zerakidze

The necessary and sufficient conditions for existence of consistent criteria in Hilbert space of measures are obtained. Let H be sets hypotheses and $\beta(H)$ – σ-algebra which contains all finite subsets of H.

Definition 1. The family of probability measures $\{\mu_h, h \in H\}$ is said to admit a consistent criterion of hypothesis if there exists even though one measurable map δ of the space (E, S) in $(H, \beta(H))$ such that $\mu_h(x : \delta(x) = h) = 1$, $\forall h \in H$.

Theorem 1. Let the family of probability measures $\{\mu_h, h \in H\}$ admits a consistent criterion of any parametric function, then the family of probability measures $\{\mu_h, h \in H\}$ is weakly separable.

Theorem 2. Let the family of probability measures $\{\mu_h, h \in H\}$ admits a consistent criterion of hypotheses then the family of probability measures $\{\mu_h, h \in H\}$ admits a consistent criterion of and parametric function and then admits unbiased criterion of any parametric function.

Theorem 3. Let M_H be a Hilbert space of measures and $M_H = \bigoplus_{i \in N} M_H(\mu_{H_i})$. For the family of probability measures $\{\mu_{H_i}, i \in N\}$ to admit a consistent criterion of hypotheses it is necessary and sufficient that the correspondence $f \to \Psi_f$ given by $\int f(x)\nu(dx) = \langle \Psi_f, \nu \rangle$, $\forall \nu \in M_H$ would be one-to-one, $f \in F(M_H)$. Where $F(M_H)$ is the set of those f, for with $\int f(x)\nu(dx)$ is defined $\forall \nu \in M_H$.

Theorem 4. Let M_H be a Hilbert space of measures and $M_H = \bigoplus_{i \in I} M_H(\mu_{H_i})$. For $\{\mu_{H_i}, i \in I\}$ to admit a consistent criterion of any parametric function it is necessary and sufficient that the family of probability measures $\{\mu_{H_i}, i \in I\}$ admits a unbiased criterion of any parametric function and the correspondence $f \to \mu_f$ given by $\int f(x)\mu_H(dx) = \langle \mu_{H_i}, \mu_H \rangle$, $\forall \mu_{H_i}, M_H$ would be one-to-one $f \in F(M_H)$.

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Gori university, Department of education, exact and natural sciences, 53 Chavchavadze str. Gori, Tbilisi, Georgia

E-mail address: lelaaleksidze@gmail.com

ON ASYMPTOTIC PROPERTIES OF ML ESTIMATORS FOR THE REGRESSION PARAMETERS UNDER CLASSIFICATION OF OBSERVATIONS

H.S. Aheyeva[1], Yu.S. Kharin[2]

Let the non-linear multiple regression be defined over probability space $(\Omega, \mathcal{F}, \mathbf{P})$:

$$Y_t = F(X_t; \theta^0) + \xi_t,\ t = 1, \ldots, n,$$

where X_t is the known regressor vector, ξ_t is a Gaussian $\mathcal{N}(0, (\sigma^0)^2)$ random variable, $\{\xi_t\}_{t=1}^n$ – i.i.d., $\alpha^0 = (\theta^0, (\sigma^0)^2) \in \mathbb{R}^{m+1}$ is an unknown parameter vector. Let $A_k = (a_{k-1}, a_k]$, $k \in \mathbf{K}$, $-\infty = a_0 < a_1 < \cdots < a_K = \infty$, be a sequence of K nonintersecting intervals. This set of intervals define a classification of the dependent variables Y_t: Y_t corresponds to the class number ν_t, if $Y_t \in A_{\nu_t}$, $\nu_t \in \mathbf{K} = \{1, \ldots, K\}$. Instead of the exact values of the dependent variables $Y_1, \ldots, Y_n$ we observe only corresponding numbers of classes $\nu_1, \ldots, \nu_n \in \mathbf{K}$. Our goal is to obtain statistical estimators of the unknown parameter vector α^0 using classified observations $\nu_1, \ldots, \nu_n \in \mathbf{K}$ and values of the regressors $X_1, \ldots, X_n$.

For estimating the vector of the parameters α^0 we use the maximum likelihood estimators $\hat{\alpha}^n$. We establish conditions under which the ML-estimators are consistent and strong consistent, as well as conditions under which asymptotic normality and unbiasedness hold [1, 2].

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[1] str. Rafieva 88a-65, Minsk, Belarus

E-mail address: helenaageeva@yahoo.ca

[2] per. Kaliningradskii 13-38, Minsk, Belarus

E-mail address: kharin@bsu.by

PERSPECTIVES ON HIGH DIMENSIONAL DATA ANALYSIS: WEIBULL CENSORED REGRESSION MODEL

S. Ejaz Ahmed

In this talk we address the problem of estimating a vector of regression parameters in the Weibull censored regression model. The main objective is to provide natural adaptive estimators that significantly improve upon the classical procedures in the situation where some of the predictors may or may not be associated with the response. In the context of two competing Weibull censored regression models (full model and candidate sub-model), we consider an adaptive shrinkage estimation strategy that shrinks the full model maximum likelihood estimate in the direction of the sub-model maximum likelihood estimate. The shrinkage estimators are shown to have higher efficiency than the classical estimators for a wide class of models. Further, we consider a LASSO type estimation strategy and compare the relative performance with the shrinkage estimators. Monte Carlo simulations reveal that when the true model is close to the candidate sub-model, the shrinkage strategy performs better than the LASSO strategy when, and only when, there are many inactive predictors in the model. Shrinkage and LASSO strategies are applied to a real data set from Veteran's administration (VA) lung cancer study to illustrate the usefulness of the procedures in practice.

Brock University, Canada
E-mail address: seahmed12@gmail.com

ON ONE PROBLEM OF OPTIMAL STOPPING WITH INCOMPLETE DATA

Petre Babilua, Besarion Dochviri, Grigol Sokhadze

The problem of optimal stopping with incomplete data is reduced to the problem of optimal stopping with complete data. Let us consider a partially observable process (θ_t, ξ_t) , $0 \leqslant t \leqslant T$, of following model

$$d\theta_t = \left[a_0(t) + a_1(t)\theta_t\right] dt + b(t)\, dw_1,$$
$$d\xi_t = d\theta_t + \varepsilon\, dw_2,$$

where $\varepsilon > 0$, w_1 and w_2 are independent Wiener process. It is assumed that θ_t is the nonobservable process and ξ_t is the observable process [1].

Define the payoffs:

$$s^0 = \sup_{\tau \in M^\theta} Eg(\tau, \theta_\tau), \quad s^\varepsilon = \sup_{\tau \in M^\xi} Eg(\tau, \theta_\tau),$$

where $g(t,x) = f_0(t) + f_1(t)x$ is the gain function, M^θ and M^ξ are the classes of stopping times with respect to the σ-algebras $\Im_t^\theta = \sigma\{\theta_s,\ s \leqslant t\}$, $\Im_t^\xi = \sigma\{\xi_s,\ s \leqslant t\}$. The problem of optimal stopping with incomplete data is reduced to the problem of optimal stopping with complete data [2].

Let $m_t = E(\theta_t |\ \Im_t^\xi)$, $\gamma_t = E(\theta_t - m_t)^2$ and define the process η_t by following equation

$$d\eta_t = \left[a_0(t) + a_1(t)\eta_t\right] dt + \frac{a_1(t)\gamma_t}{\varepsilon}\, dw_1. \tag{1}$$

Theorem 1. *Let $a_i(t)$, $f_i(t)$, $i = 0, 1$, $b(t)$ are nonrandom measurable and bounded functions. Then*

$$s^\varepsilon = \sup_{\tau \in M^\xi} Eg(\tau, m_\tau).$$

Theorem 2. *Let the process η_t is define by* (1). *Then*

1. $M^\theta = M^\eta$,
2. $s^\varepsilon = \sup_{\tau \in M^\theta} Eg(\tau, \eta_\tau)$.

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Ivane Javakhishvili Tbilisi State University, 2 University St. Tbilisi 0186, Georgia
E-mail address: petre.babilua@tsu.ge, besarion.dochviri@tsu.ge, giasokhi1@i.ua

PARAMETER ESTIMATION AND TESTING STABILITY IN A SPATIAL UNILATERAL AUTOREGRESSIVE MODEL

S. Baran[1], G. Pap[2], K. Sikolya[3]

We consider the spatial autoregressive process $\{X_{k,\ell} : k, \ell \geq 0\}$ defined as

$$X_{k,\ell} = \alpha X_{k-1,\ell} + \beta X_{k,\ell-1} + \gamma X_{k-1,\ell-1} + \varepsilon_{k,\ell},$$

and we are interested in the asymptotic behaviour of the least squares estimator (LSE) of the parameters in the unit root case, that is when the parameters are on the boundary of the domain of stability that forms a tetrahedron in $[-1,1]^3$, with vertices $\{(-1,1,1),\ (1,-1,1),\ (1,1,-1),\ (-1,-1,-1)\}$.

In the special case $\gamma = 0$ Paulauskas [4] determined the asymptotic behaviour of the variances of the process and Baran *et al.* [3] showed that the LSE of (α, β) is asymptotically normal and the rate of convergence is $n^{-3/2}$ if one of the parameters equals zero and n otherwise.

In the present work we first investigate the LSE of the stability parameter $\varrho := |\alpha| + |\beta|$ and show the asymptotic normality of the estimator with a scaling factor $n^{5/4}$, in contrast to the classical AR(p) model, where the least squares estimator of the appropriate stability parameter is not asymptotically normal [2]. The limiting distribution of the stability parameter can be applied for building unit root tests for the above spatial process.

We also consider the general model and prove that limiting distribution of the parameters (α, β, γ) is normal and the rate of convergence is n when the parameters are in the faces or on the edges of the boundary of the domain of stability, while on the four vertices, the rate is $n^{3/2}$ (see [1]).

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[1] Faculty of Informatics, University of Debrecen Kassai út 26, H–4028 Debrecen, Hungary
E-mail address: `baran.sandor@inf.unideb.hu`
[2] Bolyai Institute, University of Szeged, Aradi vértanúk tere 1, H-6720 Szeged, Hungary
E-mail address: `papgy@math.u-szeged.hu`
[3] Faculty of Informatics, University of Debrecen Kassai út 26, H–4028 Debrecen, Hungary
E-mail address: `sikolya.kinga@inf.unideb.hu`

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CROSS-CORRELOGRAM ESTIMATION OF RESPONSE FUNCTIONS OF LINEAR SYSTEMS IN SCHEMES OF ONE AND SOME INDEPENDENT SAMPLES

I.P. Blazhievska

Let $H = (H(\tau), \tau \in \mathbf{R})$ be a response function of a time-invariant linear system. The inputs $X_\Delta = (X_\Delta(t), t \in \mathbf{R}), \boldsymbol{\Delta} > \mathbf{0}$, are supposed to be real-valued stationary centered a.s. continuous Gaussian processes, that "close" to a white noise $(\Delta \to \infty)$. A response of this system on X_Δ is described by the following process

$$Y_\Delta(t) = \int_{-\infty}^{\infty} H(t-s) X_\Delta(s) ds, \ t \in \mathbf{R},$$

that is also supposed to be a.s. continuous. Using the assumption $H \in L_2(\mathbf{R})$, we consider the cross-correlogram estimates of the unknown real-valued function H.

Let us observe $n \geq 1$ independent samples $(X_\Delta^{(j)}, Y_\Delta^{(j)})$, $j = 1, ..., n$, of the pair of processes $(X_\Delta(t), t \in [0, T])$, $(Y_\Delta(t), t \in [0, T + T_1])$. We consider the estimate for H as the following sample cross-corellogram

$$\widehat{H}_{T,\Delta}^{(n)}(\tau) = \frac{1}{n} \sum_{j=1}^{n} \frac{1}{T} \int_0^T Y_\Delta^{(j)}(t+\tau) X_\Delta^{(j)}(t) dt, \ \tau \in [0, T_1],$$

and investigate the conditions of its asymptotic normality as $n, \Delta \to \infty$ in $C[0, T_1]$.

In the talk we compare the results about the cross-correlogram estimation of response function H in schemes of one sample (see, [1]) and of some independent samples. Also we construct the confidence bands for the limiting process and present many examples of H and X_Δ, for which these schemes are applicable.

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NTUU "KPI", pr. Peremogy 37, Kiev, Ukraine
E-mail address: `packsenarrion@rambler.ru`

THE RATE OF CONVERGENCE OF CORRECTED ESTIMATORS IN COX MODEL WITH MEASUREMENT ERROR

K.A. Chimisov

Consider Cox proportional hazard model where a random variable T has intensity function

$$\Lambda(t|X;\lambda,\beta) = \lambda(t)exp(\beta^\mathsf{T} X). \tag{1}$$

Here covariate X is a random vector, $\lambda(t) \in \Theta_\lambda \subset C[0,\tau]$ is the baseline hazard function and β is a parameter from $\Theta_\beta \subset \mathbb{R}^k$ which has to be estimated. We can observe only censored value of $Y := min\{T, C\}$, where censor C is distributed on $[0,\tau]$. Censorship indicator $\Delta := \mathbb{I}_{\{T\le C\}}$ is also known. X is observed with measurement error U: $W = X + U$, where U is random vector independent of $\{X, Y, C\}$ with known moment generating function $M_U(\beta) := Ee^{\beta^\mathsf{T} U}$ and zero mean. Making some restrictions on $\Theta_\lambda, \Theta_\beta$, X and U, and considering random sample $\{X_i, Y_i, \Delta_i\}, i = 1, 2, .., n$ and estimator $(\widehat{\lambda}_n, \widehat{\beta}_n)$ that maximizes the corrected log-likelihood function

$$Q_n^{cor}(\lambda,\beta) := \sum_{i=1}^{n} q^{cor}(Y_i, \Delta_i, W_i, \lambda, \beta), \tag{2}$$

where

$$q^{cor}(Y,\Delta,W,\lambda,\beta) = \Delta(\log\lambda(Y) + \beta^\mathsf{T} W) - \frac{e^{\beta^\mathsf{T} W}}{M_U(\beta)}\int_0^Y \lambda(u)\,\mathrm{d}u, \tag{3}$$

one can prove that $(\widehat{\lambda}_n, \widehat{\beta}_n)$ is strongly consistent estimator of true value (λ_0, β_0) [1]. The aim of the present work is to show that under additional assumptions it is possible to show stochastic boundedness of $\sqrt{n}\|\widehat{\beta}_n - \beta_0\|$.

The results are joint with Prof. A.G. Kukush.

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National Taras Shevchenko University, Kyiv, Ukraine
E-mail address: kirill2192@inbox.ru

THE PARAMETERS OF LINEAR ERRORS-IN-VARIABLES REGRESSION ESTIMATION USING NEGATIVE PROBABILITIES

I.A. Dobrovska

We construct the linear errors-in-variables regression parameters estimators using the concept of negative probabilities, least squares method and Monte Carlo method.

It is assumed that the "true" variables are related by the equation:

$$\eta_j = b_0^* + b_1^*\xi_j,\ j = \overline{1,n},$$

but these variables are observed with errors:

$$x_j = \xi_j + \delta_j,\ y_j = \eta_j + \epsilon_j,\ j = \overline{1,n},$$

where $\left\{\xi_j, j = \overline{1,n}\right\}$ are independent and identically distributed random variables (i.i.d.r.v.), $\left\{\epsilon_j, j = \overline{1,n}\right\}$ are i.i.d.r.v., $\left\{\delta_j, j = \overline{1,n}\right\}$ are i.i.d.r.v. It is also assumed that the following conditions are satisfied: $\left\{\xi_j, \epsilon_j, \delta_j, j = \overline{1,n}\right\}$ are independent and $E\varepsilon_1 = E\delta_1 = 0$.

There are a lot of methods of this model's parameters estimation (e.g. [2]). Using ordinary least squares method leads to inconsistent estimators of parameters. Thus we offer to construct consistent estimators using negative probabilities potential.

We formally define the notion of negative probabilities, consider the algorithm of regression parameters estimation using this notion and prove the consistency and asymptotic normality of obtained estimators. The estimators were tested on samples of moderate size with imitational modeling methods.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: dobrira@gmail.com

The author is supported by the department of probability theory, statistics and actuarial mathematics of Kyiv Taras Shevchenko National University

LIMIT THEOREMS FOR EXTREME RESIDUALS IN REGRESSION MODELS

A.V. Ivanov[1], I.K. Matsak[2]

Regression model

$$y_j = g(x_j, \theta) + \epsilon_j, \quad j = \overline{1, n},$$

is considered where ϵ_j are i.i.d.r.v.-s, $\mathbf{E}\epsilon_j = 0$, $\mathbf{E}\epsilon_j^2 = \sigma^2 < \infty$, $g : X \times \Theta^c \to R$, $X \subset R^m$, $\Theta^c \subset R^q$, is a linear or nonlinear regression function. Denote by $\hat{\theta}_n = \hat{\theta}_n(y_1, \ldots, y_n) \in \Theta^c$ the least squares estimator of unknown parameter $\theta \in \Theta$,

$$\begin{aligned} \hat{Z}_n &= \max_{1 \le j \le n} \hat{\epsilon}_j, \quad \hat{Z}_n^* = \max_{1 \le j \le n} |\hat{\epsilon}_j|, \\ \hat{\epsilon}_j &= y_j - \hat{y}_j, \quad \hat{y}_j = g(x_j, \hat{\theta}_n), \quad j = \overline{1, n}. \end{aligned}$$

Under additional assumptions on r.v.-s ϵ_j, $j \ge 1$, experiment design $\{x_j, j = \overline{1, n}\} \subset X$, $n \to \infty$, and smoothness of regression function g in nonlinear case, it is proved that distributions of the properly normed r.v.-s $\hat{Z}_n$, $\hat{Z}_n^*$ converge as $n \to \infty$ to one of the extreme distribution types [1].

One can use the limit theorems obtained for hypotheses testing on regression model adequacy.

The previous results can be extended to the case of heavy observation error tails when the expectation of ϵ_j does not exist.

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[1] National Technical University of Ukraine "Kyiv Polytechnic Institute", 37 Peremogy ave., Kyiv, Ukraine
E-mail address: alexntuu@gmail.com
[2] Taras Shevchenko National University of Kyiv, Glushkova Ave. 2, kor. 6, Kyiv, Ukraine
E-mail address: mik@unicyb.kiev.ua

PERIODOGRAM ESTIMATOR CONSISTENCY AND ASIMPTOTICAL NORMALITY OF HARMONICAL OSCILLATION PARAMETERS

A.V. Ivanov, B.M. Zhurakovskyi

Suppose we observe the random process $X(t) = A_0 cos\varphi_0 + \varepsilon(t)$, $t \ge 0$, where $A_0 \ge 0$, $0 < \underline{\varphi} < \varphi_0 < \overline{\varphi} < \infty$, $\varepsilon(t) = G(\xi(t))$, $t \in \mathbb{R}^1$, $G(x)$, $x \in \mathbb{R}^1$ is a Borel function such that $E\varepsilon(0) = 0$, $E\varepsilon^2(0) < \infty$. The random process $\xi(t)$, $t \in \mathbb{R}^1$, is a mean square continuous measurable stationary Gaussian process, $E\xi(0) = 0$, with covariance function $B(t) = L(|t|)|t|^{-\alpha}$, $\alpha \in (0, 1)$, $L(t)$, $t \ge 0$, is a nondecreasing slowly varying function at infinity, $B(0) = 1$, or $B(t) = (1 + t^2)^{-\alpha/2} \cos \psi t$, $\alpha \in (0, 1)$, $\psi > 0$. We also assume that $m\alpha > 1$, where m is Hermite rank of G.

Consider the functional $Q_T(\varphi) = \left| \frac{2}{T} \int_0^T X(t) e^{i\varphi} t dt \right|^2$. The periodogram estimator of φ_0 is said to be any random variable φ_T such that $Q_T(\varphi_T) = max_{\varphi \in [\underline{\varphi}, \overline{\varphi}]} Q_T(\varphi)$. The estimator of A_0 is defined as $A_T = Q_T^1/2(\varphi_T)$.

If all these conditions are satisfied then the vector $\left(T^{1/2}(A_t - A_0), T^{3/2}(\varphi_T - \varphi_0)\right)$ has asymptotically normal distribution, as $T \to \infty$, with zero mean and covariance matrix

$$\sigma^2 = 2\pi \sum_{j=m}^{\infty} \frac{C_j^2(G)}{j!} f^{*j}(\varphi_0) \begin{pmatrix} 2 & 0 \\ 0 & 24A_0^2 \end{pmatrix},$$

where $f^{*j}(\varphi_0) = \int_{\mathbb{R}^1} f(\lambda - \lambda_2 - \cdots - \lambda_j) \Pi_{i=2}^j f(\lambda_i) d\lambda_2 ... d\lambda_j$, $j \ge m$, is the j-th convolution of spectral density $f(\lambda)$, $\lambda \in \mathbb{R}^1$, of random process ξ, and $C_j(G) = \int_{-\infty}^{+\infty} G(x) H_j(x) \varphi(x) dx$, $H_j(x)$, $j \ge m$, are Hermite polynomials, $\varphi(x) = (2\pi)^{-1/2} e^{-x^2/2}$, $t \in \mathbb{R}^1$.

This statement generalize the results of the paper [1].

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KPI, FMF, Pr. Peremogy, 37, Kyiv, Ukraine
E-mail address: alexntuu@gmail.com, zhurak@gmail.com

ONE WAY ANALYSIS OF VARIANCE TESTS BY USING LOCATION-SCALE ESTIMATORS

D. Karagoz, T. Saracbasi

The problem of testing the homogeneity of several means in a one-way analysis of variance is one of the oldest problem in statistics. Under the classical assumptions which are normality of the errors, homogeneity of the error variance and independency; the classic F test is known to be the optimal test. However, when one or more of these assumptions are violated, the F-test becomes conservative or liberal, depending on the degree of which these assumptions are violated. By using location-scale estimators which are median/MAD and median/Qn, robust Welch, Brown-Forsythe and Modified Brown-Forsythe test statistics for the one-way ANOVA under heteroscedasticity are developed for Weibull distribution with outliers. In order to get these robust test statistics, robust estimators of mean and variance of Weibull distribution are obtained. We compare the performance of these tests and of the F-test by way of simulation with respect to their type I errors under several experimental designs: data from a skew distribution under heterogeneity and homogeneity, balanced and unbalanced sample size. The simulations results show up that the proposed robust tests have a good performance.

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Hacettepe University Department of Statistics, 06800, Beytepe Ankara, Turkey

E-mail address: deryacal@hacettepe.edu.tr, toker@hacettepe.edu.tr

ROBUSTNESS IN STATISTICAL FORECASTING

Yuriy Kharin

Many applied problems lead to an important problem of mathematical statistics – forecasting of time series. The mathematical substance of this problem is quite simple: to estimate the future value $x_{T+\tau} \in \mathbf{R}^d$ of the d-variate time series in $\tau \in \mathbf{N}$ steps ahead by $T \in \mathbf{N}$ successive observations $X = (x_1, \dots, x_T) \in \mathbf{R}^{Td}$. The majority of research was oriented to the development of optimal forecasting statistics f: $\mathbf{R}^{Td} \to \mathbf{R}^d$ that minimize the mean square risk $r(f) = \mathbf{E}\{||x_{T+\tau} - f(X)||^2\}$ for a set of simple hypothetical models $\mathbf{M}_0$, e.g., stationary time series with some known spectral density, trend models, ARIMA. In applied problems the hypothetical model $\mathbf{M}_0$ assumptions are often distorted, and this fact leads to the instability of the "optimal" forecasting statistics. Huber has proposed [1] to construct robust statistical inferences that are "weak-sensitive" w.r.t. small distortions of the hypothetical model $\mathbf{M}_0$.

This lecture presents the following results [2]-[5]: mathematical description of distortions for main hypothetical models $\mathbf{M}_0$ in forecasting; quantitative evaluation of the robustness (sensitivity analysis) under distortions for traditional forecasting statistics (based on $\mathbf{M}_0$) by the guaranteed upper risk, the risk instability coefficient and the breakdown point; new robust forecasting statistics. Computer results are given.

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Belarusian State University, Minsk, Belarus

E-mail address: kharin@bsu.by

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ESTIMATION OF REGRESSION PARAMETERS WHEN INDEPENDENT VARIABLES ARE KNOWN INEXACT

Arnold Korkhin

We will consider T pair $\left((x_t^0)^\intercal,\ y_t\right)$, $t = \overline{1,T}$, where symbol "$\intercal$" denotes the transposition; $x_t^0 \in \mathbb{R}^n$, $y_t \in \mathbb{R}^1$, $t = \overline{1,T}$. At that vectors x_t^0, $t = \overline{1,T}$ are unknown. These values and y_t, $t = \overline{1,T}$, connected by relationship

$$Y = X^0\alpha^0 + \varepsilon, \tag{1}$$

where $T \times n$-matrix X^0 has the rows $(x_t^0)^\intercal$, $t = \overline{1,T}$; $\alpha^0 \in \mathbb{R}^n$ is regression parameter (unknown); $Y = [y_t]$, $t = \overline{1,T}$; $\varepsilon \in \mathbb{R}^T$ has the uncorrelated components ε_t, $t = \overline{1,T}$.

As a result T observations the pair (Y, X^*) is got, where $T \times n$-matrix X^* of values of independent variables. Let X^0 in (1) satisfies to the restriction

$$X^0 - \Delta \le X^* \le X^0 + \Delta, \tag{2}$$

where $X^* = [x^*_{ti}]$, $\Delta = [\delta_{ti}]$, $t = \overline{1,T}$, $i = \overline{1,n}$.Values δ_{ti} are assumed by known. In particular, δ_{ti} can be equal 10^{k_i} for all t and $i = \overline{1,n}$, where k_i is the last significant digit in numbers x^*_{ti}, $t = \overline{1,T}$.

We will find estimation α^0 as solution of the problem

$$\|Y - X\alpha\|^2 \Longrightarrow \min,\ X^* - \Delta \le X \le X^* + \Delta. \tag{3}$$

In (2) a minimum is searched on $\alpha \in \mathbb{R}^n$ and X.

The algorithm of decision of the problem (2) is described and properties of the got estimations are analyzed. A next example is described.

Dependence between mechanical property of carbon steel and its analysis for the t-th observation is given by

$$y_t = \left(x^0_t\right)^{\mathsf{T}} \alpha^0 + \varepsilon_t,\ i = \overline{1,T}, \tag{4}$$

where $\alpha^0 \in \mathbb{R}^n$.

Components of x^0_t : first is unit, the second, third and fourth components are maintenance of carbon, manganese and silicon in steel (in percents).

The components of x^0_t , except for the first, are measured with errors. Because often regression model of dependence of mechanical property from analysis of steel has the small coefficient of determination R^2, and the signs of estimations of regression parameters are mismatch to physical sense. The evaluation according to (2) increased R^2 from 0.316 to 0.750 and gave the right signs of estimations of regression parameters.

National Mining University, Economical Cybernetics and Information Technology, 19 Karl Marx Avenue, Dnepropetrovsk, Ukraine
E-mail address: korkhin@mail.ru

THE RATE OF CONVERGENCE OF HURST INDEX ESTIMATE FOR STOCHASTIC DIFFERENTIAL EQUATION INVOLVING FRACTIONAL BROWNIAN MOTION

K. Kubilius[1], Yu.S. Mishura[2]

We consider a stochastic differential equation involving a pathwise integral with respect to fractional Brownian motion. The estimates for the Hurst parameter are constructed according to first- and second-order quadratic variations of observed values of the solution. The rate of convergence of these estimates to the true value of a parameter is established when the diameter of interval partition tends to zero.

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[1] Vilniaus university Institute of Mathematics and Informatics, Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail address: kestutis.kubilius@mii.vu.lt
[2] National Taras Shevchenko University of Kyiv, Volodymyrska 64, 01601 Kiev, Ukraine
E-mail address: myus@univ.kiev.ua

RADIATION RISK ESTIMATION UNDER UNCERTAINTY IN DOSES

A.G. Kukush

With a binary response Y, consider the *dose-response model* with $P(Y=1|D) = R(1+R)^{-1}$, $R = \lambda_0 + EAR \times D$, where λ_0 is the *baseline incidence rate* and EAR is *the excess absolute risk per gray.* The calculated thyroid dose of a person i is $D_i^{mes} = f_i^{mes} Q_i^{mes} / M_i^{mes}$. Here, Q_i^{mes} is the measured content of radioiodine in the thyroid gland of person i at time t_{mes}, M_i^{mes} is the estimate of the thyroid mass, and f_i^{mes} is the normalizing multiplier. Q_i is measured with *classical* additive error, so that $Q_i^{mes} = Q_i^{tr} + \sigma_{Q,i}^{mes}\gamma_i$, where $\{\gamma_i\}$ form Gaussian white noise and standard deviation $\sigma_{Q,i}^{mes}$ is given. M_i is measured with *Berkson* multiplicative error V_i^M, so that $M_i^{tr} = M_i^{mes} V_i^M$. Here Q_i^{tr} is the true content of radioactivity in the thyroid gland, and M_i^{tr} is the true value of the thyroid gland. f_i is also contaminated with error. Introduce a new latent variable $\overline{D}_i^{tr} = F_i^{mes} Q_i^{tr}$.The model of observations can be transformed to the form $D_i^{mes} = \overline{D}_i^{tr} + \varepsilon_i, D_i^{tr} = \overline{D}_i^{tr} V_i^D$.

Here, ε_i are Gaussian classical additive errors with known variances, and V_i^D are log-normal Berkson multiplicative errors with known parameters. The variables $\overline{D}_i^{tr}$, ε_i, $V_i^D, i = 1, \dots, N$ are mutually independent, and $\{\overline{D}_i^{tr}\}$ are assumed to be i.i.d. *log-normally* distributed with unknown parameters. By means of Regression Calibration, Method of Moments, and by properly tuned SIMEX Method we investigate the influence of measurement errors in dose on the estimates of λ_0 and EAR. Simulation study is based on a real sample of thyroid doses from epidemiological data. The true risk parameters are given by the values of earlier epidemiological studies, and then the binary response variables are simulated according to the dose-response model.

The results are joint with Prof. R.J. Carroll and Dr. A. Bouville (USA) and Prof. I. Likhtarov and Doctors S. Shklyar, S. Masiuk, M. Chepurny and L. Kovgan (Kyiv).

Taras Shevchenko National University of Kyiv, Faculty of Mechanics & Mathematics, Volodymyrska st. 64, 01601, Kyiv, Ukraine
E-mail address: Alexander_ kukush@univ.kiev.ua

ON A MULTIDIMENSIONAL GENERAL BOOTSTRAP FOR EMPIRICAL ESTIMATOR OF CONTINUOUS-TIME SEMI-MARKOV KERNELS WITH APPLICATIONS

Salim Bouzebda, Nikolaos Limnios

In the present paper, a general notion and results of bootstrapped empirical estimators of the semi-Markov kernels and of the conditional transition probabilities for semi-Markov processes with countable state space, constructed by exchangeably weight- ing sample, are introduced. Our proposal provides a unication of bootstrap methods in the semi-Markov setting including, in particular, the Efron's bootstrap. Asymptotic properties of these generalized bootstrapped empirical distributions are obtained, under mild conditions, by mean of the martingale approach. As a by-product, we obtain some new results for the weak convergence of semi-Markov processes which are of their own interest. We apply these general results on several statistical problems such as condence bands and the goodness of t test where the limiting distribution is derived under the null hypothesis. Finally, we introduce the quantile estimators and their bootstrapped versions in the semi-Markov framework and we establish their limiting laws by using the functional delta methods.

Laboratoire de Mathématiques Appliquées, Université de Technologie de Compiègne, France
E-mail address: nikolaos.limnios@utc.fr

SEMIPARAMETRIC STATISTICS OF MIXTURES WITH VARYING CONCENTRATIONS

R. Maiboroda, O. Sugakova, A. Doronin

Consider observations of subjects belonging to M different subpopulations (components). The component to which a subject belongs is unknown, so the distribution of an observed feature ξ_j of the j-th observed subject is a mixture of components distributions F_m, $m = 1, \dots, M$, i.e.

$$\mathsf{P}\{\xi_j \in A\} = p_j^1 F_1(A) + p_j^2 F_2(A) \cdots + p_j^M F_M(A),$$

where p_j^m is is the probability to observe a subject from the m-th component of the mixture in the j-th observation. We assume a parametric model for the first component of the mixture, say $F_1(A) = F(A, \theta)$, where F known, $\theta \in \Theta \subseteq \mathbf{R}^d$ is an unknown parameter. $F_2, \dots, F_m$ are unknown.

Consider a generalized estimating equation (GEE) for θ of the form

$$\hat{g}_n(t) = \frac{1}{n} \sum_{j=1}^{n} a_{j:n} g(\xi_{j:n}, t) = 0, \tag{1}$$

where $g(x,t) = (g_1(x,t), \dots, g_d(x,t))^T$ with $\int g_i(x,t)F(dx,t) = 0$, $i = 1, \dots d$, $a = (a_{1:n}, \dots a_{n:n})^T$ is the minimax weight defined by $a_{j:n} = \sum_{k=1}^{M} \gamma_{ik}^{+} p_j^k$, $\Gamma_n^{+} = (\gamma_{ik}^{+})_{i,k} = 1^M$ is the Moore-Penrose inverse to the matrix $\Gamma_n = \left(\frac{1}{n}\sum_{j=1}^{n} p_j^i p_j^k\right)_{i,k=1}^{M}$.

Any statistics $\hat{\vartheta}_n$ which satisfies (1) a.s. is a GEE-estimate for ϑ.

Denote $\alpha_i = \lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n} (a_{j:n})^2 p_j^i$, $\alpha_{ik}^{*} = \lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n} (a_{j:n})^2 p_j^i p_j^k$, $\bar{g}^i = \int g(x,\theta)F_i(dx)$, $Z = Z(g) = \sum_{i=1}^{M} \alpha_i \int g(x,\theta) g^T(x,\theta) F_i(dx,\theta) - \sum_{i,k=2}^{M} \alpha_{ik}^{*} \bar{g}^i \bar{g}^k$.

Then under suitable assumptions $\sqrt{n}(\hat{\theta}_n - \theta) \Rightarrow N(0, S(g))$, where $S(g) = V^{-1} Z V^{-T}$.

We discuss adaptive choice of g to minimize the dispersion matrix $S(g)$.

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Kyiv National University, Ukraine

E-mail address: mre@univ.kiev.ua

DURBIN-KNOTT COMPONENTS OF THE CRAMER-VON MISES TEST

G. Martynov

Cramer-von Mises statistic can be represented in the form

$$\omega_n^2 = n\int_0^1 \psi^2(t)(F_n(t)-t)^2 dt = \int_0^1 \xi_n^2(t)dt = \sum_{i=1}^{\infty} V_{i,n}^2 = \sum_{i=1}^{\infty} \frac{D_{i,n}^2}{\lambda_i},$$

where $V_{i,n} = \int_0^1 \xi_n(t)\varphi_i(t)dt$ are the Durbin-Knott components of the empirical process $\xi_n(t)$. Here, λ_i and $\varphi_i(z)$ are the eigenvalues and eigenfunctions of the limit Gauss process $\xi(t)$. The values $D_{i,n} = \sqrt{\lambda_i} V_{i,n}$ are the normalized components such that $ED_{i,n} = 0$, $ED_{i,n}^2 = 1$, $i = 1, 2, \dots$. Let $d_1, d_2, \dots$ be a sequence such that $1/d_1 + 1/d_2 + \dots < \infty$. We will consider the new statistic form $\omega_n^{*2} = \sum_{i=1}^{\infty} D_{i,n}^2/d_i$. This is the statistic with double weighting. The first weighting is based on the weight function $\psi(t)$ and another weighting is based on the sequence $d_1, d_2, \dots$. We consider the representation of ω_n^{*2} in his limit form

$$\omega^{*2} = \int_0^1\int_0^1 \xi(s)W(s,z)\xi(z)dsdz, \quad W(s,z) = \sum_{i=1}^{\infty} \frac{\lambda_i}{d_i}\varphi_i(s)\varphi_i(z).$$

There, $W(s,z)$ is the positively defined function and d_i/λ_i must tend to ∞ more quickly as i. Another transformation is

$$\omega^{*2} = \int_0^1 \left[\int_0^1 \xi(s)W^{<1/2>}(s,t)ds\right]^2 dt, \quad W(s,z) = W^{<1/2>}(s,z) = \sum_{i=1}^{\infty} \sqrt{\frac{\lambda_i}{d_i}}\varphi_i(s)\varphi_i(z).$$

The ratio d_i/λ_i must tend to ∞ more quickly as i^2. We will consider some properties of new Cramer-von Mises statistics.

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Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Bolshoy Karetny per. 19, Moscow, 127994, Russia

E-mail address: martynov@iitp.ru

TESTING HYPOTHESES IN A BANACH SPACE OF MEASURES

M. Mumladze, Z. Zerakidze

The necessary and sufficient conditions for existence of consistent criteria in Banach space of measures are obtained.

Let H be sets hypotheses and $\beta(H)$ – σ-algebra which contains all finite subsets of H.

Definition 1. The family of probability measures $\{\mu_h, h \in H\}$ is said to admit a consistent criterion of hypothesis if there exists even though one measurable map δ of the space (E,S) in $(H, \beta(H))$ such that $\mu_h(x : \delta(x) = h) = 1$, $\forall h \in H$.

Theorem 1. The family of probability measures $\{\mu_h, h \in H\}$ admits a consistent criterion of hypotheses if and only if when the probability of mistakes of all kind is zero.

Theorem 2. Let the family of probability measures $\{\mu_h, h \in H\}$ admits a consistent criterion of hypotheses then the of the family of probability measures $\{\mu_h, h \in H\}$ is strongly separable.

Theorem 3. Let M_B be a Banach space of measures and $M_B = \bigoplus_{i\in I} M_B(\mu_{H_i})$. For the family of probability measures $\{\mu_{H_i}, i \in I\}$ to admit a consistent criterion of any parametric function it is necessary and sufficient that the family of probability measures $\mu_h \in M_B$ admits a unbiased criterion of any parametric function and the correspondence

$f \to \mu_{h_f}$ given by by the equality $\int f(x)\mu_H(dx) = l_f(\mu_h),$, $\forall \mu_{H_i}, M_B$ would be one-to-one. Here $l_f(\mu_h)$ is a linear continuous functional on. M_B.

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Gori university, Department of education, exact and natural sciences, 53 Chavchavadze str., Tbilisi, Georgia

E-mail address: mmumladze@mail.ru

ON THE CRAMER–RAO INEQUALITY IN THE HILBERT SPACE

Elizbar Nadaraya, Grigol Sokhadze, Petre Babilua

We will investigate the problem of estimation of a random value $X = X(\theta;\omega)$. Let $\{\chi, \mathfrak{R}, (P(\theta;\cdot), \theta \in \Theta)\}$ be the corresponding statistical structure where χ is a separable real Hilbert space and Θ is a smooth manifold imbedding in an other Hilbert space Ξ. For a vector $\vartheta \in \Xi$ and fixed $A \in \mathfrak{B}$, we consider the derivative of the function $\tau(\theta) = P(\theta; A)$ along $\vartheta - d_\theta P(\theta; A)\vartheta$. This is an alternating measure, absolutely continuous along ϑ. $\ell_\theta(x;\vartheta) = \frac{d_\theta P(\theta;dx)\vartheta}{P(\theta;dx)}$ is called the logarithmical derivative of the $P(\theta;\cdot)$ with respect to the parameter. Also, it will be assumed that the family $(P(\theta;\cdot), \theta \in \Theta)$ is smooth with respect to the set argument in a sense of the existence of a logarithmical derivative (according to Fomin [1]) $\beta_\theta(x;h)$ for a sufficiently rich class of vectors $h \in \chi$.

Under the initial conditions of regularity there exists a natural relation between ℓ_θ and β_θ:

$$\ell_\theta(x;\vartheta) = -\beta_\theta(x, K_{\theta,\vartheta}(x)),$$

where

$$K_{\theta,\vartheta}(x) = E\left\{\frac{dx}{d\theta}X(\theta)\vartheta \middle|\ X(\theta) = x\right\}.$$

Let $T(X) : \chi \to R$ be a statistic and $g(\theta) = E_\theta(T(X))$. The Cramer–Rao inequality

$$\operatorname{Var} T(x) \geq \frac{(g'_\theta(\theta))^2}{E_\theta \ell^2_\theta(X;\vartheta)}$$

is valid.

Such consideration makes it possible to formulate the maximal likelihood princip in a Hilbert space and prove the properties of estimate consistency and asymptotical normality. For Gaussian spaces, the consideration reduces to the application of the Maliavin calculus.

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Ivane Javakhishvili Tbilisi State University, 3 University St. Tbilisi 0186, Georgia

E-mail address: elizbar.nadaraya@tsu.ge, grigol.sokhadze@tsu.ge, petre.babilua@tsu.ge

LEAST SQUARES ESTIMATOR IN LINEAR REGRESSION WITH LONG-RANGE AND WEAK DEPENDENT REGRESSORS

Igor V. Orlovsky

Consider a regression model

$$X(t) = \sum_{i=1}^{q} \theta_i z_i(t) + \varepsilon(t),\ t \in [0,T], \tag{1}$$

where $z_i(t) = a_i(t) + y_i(t),\ i = \overline{1,q}$, are regressors which observed with errors, $a_i :\ [0,\infty) \to \mathbb{R}^1,\ i = \overline{1,q}$, are some non random continuous functions, $\theta = (\theta_1, ..., \theta_q) \in \mathbb{R}^q$ is unknown vector parameter, $y_i(t),\ i = \overline{1,q},\ \varepsilon(t),\ t \in \mathbb{R}^1$, are independent real measurable mean-square continuous stationary Gaussian processes with zero mean.

Definition 1. Least squares estimator (lse) of unknown parameter θ obtained from observation $\left\{X(t), z_i(t),\ i = \overline{1,q},\ t \in [0,T]\right\}$ is said to be any random vector $\widehat{\theta}_T = \widehat{\theta}_T\left(X(t), z_i(t),\ i = \overline{1,q},\ t \in [0,T]\right)$ having the property

$$Q_T(\widehat{\theta}_T) = \inf_{\tau \in \mathbb{R}^q} Q_T(\tau),\ Q_T(\tau) = \int_0^T \left[X(t) - \sum_{i=1}^{q} \tau_i z_i(t)\right]^2 dt.$$

Consistency and asymptotic normality of lse of unknown parameter of model (1) in the case when functions $a_i(t),\ t \in \mathbb{R}^1,\ i = \overline{1,q}$, are constants and processes $y_i(t),\ i = \overline{1,q},\ \varepsilon(t),\ t \in \mathbb{R}^1$, are weakly dependent is considered in the book of A.Ya. Dorogovtsev [1]. Sufficient conditions of consistency and asymptotic normality of unknown parameter θ lse with processes $y_i(t),\ i = \overline{1,q},\ \varepsilon(t),\ t \in \mathbb{R}^1$, satisfying long-range or weak dependent condition are presented in the talk.

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Department of Analysis and Probability, Faculty of Physics and Mathematics, National Technical University of Ukraine "KPI", Peremogy prosp. 37, Kyiv, Ukraine
E-mail address: i.v.orlovsky@gmail.com

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ASYMPTOTIC PROPERTIES OF CORRECTED SCORE ESTIMATOR IN AUTOREGRESSIVE MODEL WITH MEASUREMENT ERRORS

D.S. Pupashenko

We consider autoregressive model with errors in variables, where control sequence is normally distributed. For the main sequence, two cases are considered: (a) main sequence has stationary initial distribution, or (b) initial distribution is arbitrary, independent of the control sequence and has the fourth moment. Here the elements of the main sequence are not observed directly, but surrogate data that include a normally distributed additive error is observed. Errors and main sequence are assumed to be mutually independent.

We estimate unknown parameter using the Corrected Score method [2] and in both cases prove the strict consistency and the asymptotic normality of the estimator. To prove asymptotic normality we use the theory of the strong mixing sequences of random variables [1]. Finally we compare the efficiency of the Least Squares (naive) estimator and the corrected-score method's estimate in the forecasting problem, and as a result, we get that the naive estimate gives the better forecast.

The results are joint with Prof. A. Kukush.

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Hochschule Furtwangen, Department of Mechanical and Process Engineering, Jakob-Kienzle-StraЯe 17, 78054 Villingen-Schwenningen, Germany
E-mail address: pud@hs-furtwangen.de

ROBUST KALMAN TRACKING AND SMOOTHING WITH PROPAGATING AND NON-PROPAGATING OUTLIERS

D.S. Pupashenko[1], P. Ruckdeschel[2], B. Spangl[3]

A common situation in filtering where classical Kalman filtering does not perform particularly well is tracking in the presence of propagating outliers. This calls for robustness understood in a distributional sense, i.e.; we enlarge the distribution assumptions made in the ideal model by suitable neighborhoods. Based on optimality results for distributional-robust Kalman filtering from , we propose new robust recursive filters and smoothers designed for this purpose. We apply these procedures in the context of a GPS problem arising in the car industry. To better understand these filters, we study their behavior at stylized outlier patterns (for which they are not designed) and compare them to other approaches for the tracking problem. Finally, in a simulation study we discuss efficiency of our procedures in comparison to competitors. Particular focus goes to the implementation of these procedures to R.

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[1] Hochschule Furtwangen, Department of Mechanical and Process Engineering, Jakob-Kienzle-StraЯe 17, 78054 Villingen-Schwenningen, Germany
E-mail address: pud@hs-furtwangen.de

[2] Fraunhofer ITWM, Department of Financial Mathematics, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany
E-mail address: peter.ruckdeschel@itwm.fraunhofer.de

[3] University of Natural Resources and Applied Life Sciences, Institute of Applied Statistics and Computing, Gregor-Mendel-Str. 33, 1180 Wien, Austria
E-mail address: bernhard.spangl@boku.ac.at

ESTIMATORS OF COMPONENTS OF MIXTURES WITH VARYING CONCENTRATIONS BY CENSORED OBSERVATIONS: A COUNTING PROCESS APPROACH

Anton Ryzhov

Consider a general random censorship model for mixtures with varying concentrations [1]. Namely, for each $n = 1, 2, \ldots$ let T_{ij}^n and $U_{ij}^n, i = 1, \ldots, n_j, j = 1, \ldots, N$ are $2n$ independent positive random variables, T_{ij}^n or U_{ij}^n almost surely finite for each i, j and n. Let T_{ij}^n has (sub)-distribution function $F_j^n(t) = P\{T_{ij}^n \le t\}$ and U_{ij}^n has (sub)-distribution function $L_{ij}^n = P\{U_{ij}^n \le t\}, t \ge 0$. The observable random variables X_{ij}^n and η_{ij}^n are defined by $X_{ij}^n = \min(T_{ij}^n, U_{ij}^n)$ and $\eta_{ij}^n = I\{X_{ij}^n = T_{ij}^n\}$, where $I\{A\}$ is indicator function of a set A. We assume that there exist

m different populations $\Omega_1, \Omega_2, \ldots, \Omega_m, 1 \leq m \leq N$, with (sub)-distribution functions $H_1^n(t), H_2^n(t), \ldots, H_m^n(t),\ t \geq 0$ such that for each j and n

$$F_j^n(t) = w_1^{(j)} H_1^n(t) + \cdots + w_m^{(j)} H_m^n(t),\ t \geq 0,$$

where $w_l^{(j)}$ is probability for subject from the j-th sample to belong to population $\Omega_l, l = 1, \ldots, m$, $\sum_{l=1}^m w_l^{(j)} = 1,\ 0 \leqslant w_l^{(j)} \leq 1$.

In present work we generalize the results on adaptive estimators of $H_1^n(t), \ldots, H_m^n(t)$, obtained in [1] for the case of absolutely continuous (sub)-distribution functions $F_j^n(t) = F_j(t),\ L_{ij}^n(t) = L_j(t),\ t \geq 0$, to the case of possibly discontinuous functions, possessing dependance on the sample size n, using counting process approach from [2].

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Department of General Mathematics, Kyiv National Taras Shevchenko University, Volodymyrska str. 64, 01033 Kyiv, Ukraine

E-mail address: ryzhov@univ.kiev.ua

PRESERVATION RESULTS FOR NWUCA CLASS OF LIFE DISTRIBUTIONS UNDER RELIABILITY OPERATIONS

G. Saidi[1], A. Aissani[2]

Several extensions of the NBU (New Better than Used) class of life distributions have been proposed in the literature. In this communication, we discuss the dual class of the NBUCA (New Better than Used in Increasing Convex Average Order) defined recently by Ahmad & al. (2006). The aging properties defining this nonparametric class are based on comparing between the life distribution of a new unit to the remaining life or a used unit in the increasing convex average order. We describe this class by comparing the life distribution of a new unit to the mean residual life function of the asymptotic remaining survival time of the unit under repeated perfect repairs. We prove that the NWUCA class is not closed under the formation of coherent systems and convolution. We prove also that the NWUCA property is not preserved under arbitrary mixtures and a stronger hypothesis (of non-crossing mixtures and a common mean) is required. Finally, we prove that the moment generating function for NBUCA distribution provided that the life has finite two moments.

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[1] National High School of Statistic and Applied Economics, Street n 11 Doudou Mokhtar, Ben-Aknoun, Algiers, Algeria

E-mail address: ghsaidi@yahoo.fr

[2] University of Science and Technology Houari Boumediene, El-Alia, Bab-Ezzouar, Algiers, Algeria

E-mail address: amraissani@yahoo.fr

LIMIT THEOREMS FOR THE INTEGRALS OF THE SQUARED PERIODOGRAM AND STATISTICAL APPLICATIONS

L. Sakhno

In the talk, we present limit theorems for the integrals of the squared periodogram for stationary stochastic processes. The limit theorems obtained are applied for statistical parameter estimation in the spectral domain: a class of minimum contrast estimators is introduced, conditions for consistency and asymptotic normality of the estimators are stated.

Kyiv National Taras Shevchenko University, Kyiv, Ukraine

E-mail address: lms@univ.kiev.ua

CORRECTED $T(q)$-LIKELIHOOD ESTIMATOR IN EXPONENTIAL MEASUREMENT ERROR MODEL

A.V. Savchenko

Consider exponential errors-in-variables model $f(y \mid \lambda) = \lambda \exp(-\lambda y),\ y \geqslant 0,\ \lambda = \exp(\beta_0 + \beta_1 \xi),\ x = \xi + \delta$, where ξ, ε, δ are independent, ε and δ are centered errors, $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$, σ_δ^2 is known, the distribution of ξ and regression parameters β_0, β_1 are unknown. Let $(y_i,\ \xi_i, x_i), i = 1, \ldots, n$ be independent copies of the model. We observe $(y_i, x_i), i = 1, \ldots, n$. For $u > 0, q > 0$ introduce the Box-Cox transformation

$T(q,u) = \dfrac{u^{1-q}-1}{1-q}$, if $q \neq 1$, and $T(q,u) = \log u$, if $q = 1$. Let $f(y,\xi,\beta)$ be the conditional density of y given ξ, where $\beta = (\beta_0, \beta_1)^t$. The $T(q)-$likelihood score is defined as $S^{(q)}(y,\xi,\beta) = \frac{\partial}{\partial \beta^t} T\left(q, f(y,\xi,\beta)\right)$. We correct this score for measurement error and construct the corrected score $S_C^{(q)}$ such that $\mathsf{E}\left[S_C^{(q)}(y,x,\beta) \mid y,\xi\right] = S^{(q)}(y,\xi,\beta)$, a.s. The function $S_C^{(q)}$ can be expanded as $S_C^{(q)} = \sum\limits_{m=0}^{\infty} S_{C,m}^{(q)}$. The corrected $T(q)-$likelihood estimator $\widehat{\beta}_n$ is a solution to the equation $\sum\limits_{i=1}^{n} \sum\limits_{m=0}^{M_n} S_{C,m}^{(q)}(y_i, x_i, \beta) = 0, \beta \in \Theta \subset \mathbb{R}^2$, where $M_n, n \geqslant 1$ is nonrandom sequence, $M_n \to +\infty$ as $n \to \infty$.

Under the absence of measurement error δ, the $T(q)$ -likelihood estimator was considered in [2]. When $q < 1$ the role of higher value observations is reduced.

Theorem. *Let the following conditions hold: 1. $q = q_n$, where $0 < q_n \leqslant 1, n \geqslant 1$, and $\sqrt{n}(1-q_n) \to 0$, as $n \to \infty$. 2. The parameter set Θ is a given compact set, and the true value of β is an interior point of Θ. 3. $\mathsf{D}\xi \neq 0$, and there exists $K > 0$ such that $|\xi| \leqslant K$ a.s., where K is an unknown constant. Then* $\sqrt{n}(\widehat{\beta}_n - b) \xrightarrow{\mathrm{d}} \mathcal{N}(0, V^{-1}\Sigma V^{-1})$, *as $n \to \infty$, where* $V = \left(\begin{smallmatrix} 1 & \mathsf{E}\xi \\ \mathsf{E}\xi & \mathsf{E}\xi^2 \end{smallmatrix}\right)$, $\Sigma = Cov\left(S_C^{(1)}(y,x,b)\right)$.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymyrska st. 64, 01601, Kyiv, Ukraine

E-mail address: nebulous@bigmir.net

ASYMPTOTIC UNIQUENESS OF M-ESTIMATOR OF NONLINEAR REGRESSION MODEL PARAMETER

I.N. Savych

Nonlinear regression model $X(t) = g(t,\theta) + \varepsilon(t)$, $t \geq 0$, is considered where $g(t,\theta) \in C\left([0,+\infty) \times \Theta^c\right)$, $\Theta \subset \mathbb{R}^q$ is an open bounded set, $\varepsilon(t) = G(\xi(t))$, $t \in \mathbb{R}$, with Borel function $G(x)$, $x \in \mathbb{R}$, such that $\mathsf{E}\,\varepsilon(0) = 0$, $\mathsf{E}\,\varepsilon^8(0) < \infty$. Random process $\xi(t)$, $t \in \mathbb{R}$, is mean square continuous measurable stationary Gaussian process, $\mathsf{E}\,\xi(0) = 0$, $\mathsf{E}\,\xi(t)\xi(0) = B(t) = \sum_{j=0}^{r} A_j B_{\alpha_j,\chi_j}(t)$, $t \in \mathbb{R}$, $r > 0$ where $A_j > 0$, $\sum_{j=1}^{r} A_j = 1$, $B_{\alpha_j,\chi_j}(t) = \frac{\cos(\chi_j t)}{(1+t^2)^{\frac{\alpha_j}{2}}}$, $j = \overline{0,r}$, $0 = \chi_0 < \chi_1 < ... < \chi_r < +\infty$, $\alpha_j \in (0,1)$.

M- estimator of unknown parameter $\theta = (\theta_1, ..., \theta_q) \in \Theta$ is said to be any random vector $\hat{\theta}_T = \hat{\theta}_T\left(X(t), t \in [0,T]\right) \in \Theta^c$ such that

$$Q_T(\hat{\theta}_T) = \min_{\tau \in \Theta^c} Q_T(\tau), Q_T(\tau) = \int_0^T \rho\left(X(t) - g(t,\tau)\right) dt,$$

where loss function $\rho : \mathbb{R} \to \mathbb{R}$ is continuous.

Let the regression and loss functions are differentiable. Then M- estimator satisfies the system of normal equations $\nabla Q_T(\tau) = 0$. Denote by $I_T(\theta) = \left(\frac{1}{T}\int_0^T g'_{\theta_i}(t,\theta) g'_{\theta_l}(t,\theta) dt\right)_{i,l=1}^{q}$, $\theta \in \Theta$ the matrix for which $\lambda_{\min}(I_T(\theta)) \geq \lambda_0 > 0$ as $T \to \infty$.

Assume that for any $r > 0$ $P\left(\|T^{-\frac{1}{2}} d_T(\theta)(\hat{\theta}_T - \theta)\| \geq r\right) \to 0$ as $T \to \infty$.

Under additional conditions imposed on derivatives of loss function and if some assumptions on the increasing of the first and second partial derivatives of the function $g(t,\tau)$ with respect to $\tau = (\tau_1, ..., \tau_q)$ are fulfilled, then for any $\varepsilon > 0$ there exists $T_0 = T_0(\varepsilon)$ such that for $T > T_0$ the system of normal equations has an unique solution with probability $(1-\varepsilon)$ at least. This result generalizes the corresponding result stated in [1].

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National Technical University of Ukraine "Kiev Polytechnical Institute", Physical and Mathematical faculty, avenue Peremogy 37, Building 7, Kiev, Ukraine

E-mail address: sim_ka@i.ua

HOW TO TEST THE HYPOTHESIS CONCERNING THE FORM OF COVARIANCE FUNCTION OF GAUSSIAN STOCHASTIC PROCESS

M.P. Sergiienko

We consider stochastic processes that belong to the Orlicz space of random variables. Method of majorizing measures is used to construct the criterion for testing the hypothesis concerning the form of covariance function of Gaussian stationary stochastic process.

Criterion 1. *Let $X = \{X(t), t \in [0,T]\}$ be Gaussian centered stationary process. We use correlogram*

$$\widehat{B}_T(\tau) = \frac{1}{T}\int_0^T X(t)X(t+\tau)dt, \quad 0 \le \tau \le S$$

as an estimate of the covariance function of this process.

For a given level α, $0 \le \alpha \le 1$ we find x_α such that

$$f(x_\alpha) = \alpha,$$

where $f(x) = 2\exp\left\{-\ln(1+\frac{1}{A(S)})\frac{x}{C}\right\}$. Here function $A(\cdot)$ and constant C are known.

The hypothesis

$$H : \left\{\mathbb{E}X(t)X(t+\tau) = B(\tau)\right\}$$

is accepted if

$$\sup_{0\le\tau\le S}\left|B(\tau) - \widehat{B}_T(\tau) - \frac{1}{S}\int_0^S B(u)du + \frac{1}{S}\int_0^S \widehat{B}_T(u)du\right| > x_\alpha,$$

and rejected otherwise.

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Taras Shevchenko Kyiv University, Kyiv, Ukraine

E-mail address: koljasergienko@gmail.com

FINITE SAMPLE PERFORMANCE OF AN EFFICIENT FORECAST IN LINEAR MODELS

Shalabh[1], Anoop Chaturvedi[2]

Forecasting in an important aspect in linear regression analysis. Generally, the predictors are constructed and their performance is analyzed when they are employed for predicting either an average value of response variable or an actual value of the response variable. In many applications, the simultaneous forecast of actual and average value is desirable. This can be achieved by using a target function, see Rao et al. (2008). The present paper considers improved forecasts for the response variable in multiple linear regression model based on minimum risk approach. For studying the finite sample performance of the forecasts, exact expressions for first two moments have been derived. Since the expressions for finite sample moments are intricate enough to draw any clear inference, large non-centrality parameter approximations for the bias and MSE have also been obtained. The results of a numerical evaluation have been presented and values of bias and MSE are tabulated for different parameter values.

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[1] Department of Mathematics and Statistics, Indian Institute of Technology Kanpur, Kanpur - 208 016, INDIA

E-mail address: shalab@iitk.ac.in

[2] Department of Statistics, Allahabad University, Allahabad-211002, INDIA

E-mail address: anoopchaturv@gmail.com

QUADRATIC ESTIMATES OF SUBPOPULATIONS MEANS BY SURVEYS WITH ANONYMOUS COMPONENT

A.M. Shcherbina

Let we have a sample of K groups of objects. Each object belongs to one of two classes. Group i consists of $N_i^{(1)}$ objects of first class and $N_i^{(2)}$ objects of second class. With each object we observe some characteristic X. It's distribution depends only on the class of the object. Thus we have a sample $\{X_{ij},\ i = 1,\ldots,K,\ j = 1,\ldots,N_i^{(1)}+N_i^{(2)}\}$. We want to estimate mean values of the characteristic X in both classes.

Consider the following statistic for the group i:

$$S_i = \left(\sum_{j=1}^{N_i} X_{ij}, \sum_{j\ne k} X_{ij}X_{ik}\right)^T.$$

Let us denote it's mathematical expectation by $f(N_i,\mu)$, and covariance matrix by $\Sigma(N_i,\gamma)$, where μ and σ^2 are mathematical expectations and variances for classes, and $\gamma = (\mu,\sigma^2)$. Then we consider functional

$$Q_K(m,\gamma) = \sum_{i=1}^{K}(S_i - f(N_i,m))^T\left(\Sigma(N_i,\gamma)\right)^{-1}(S_i - f(N_i,m)).$$

We use the point of minimum of this functional with respect to m as an estimate of μ. Also, we consider the minimisation of the functional $Q_K(m,\hat{\gamma}_K)$, where $\hat{\gamma}_K$ is some estimate of γ. We analyze asymptotic properties of these estimates when number of groups K tends to infinity.

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Kiev National Taras Shevchenko University, Ukraine

E-mail address: artshcherbina@gmail.com

IDENTIFIABILITY OF BERKSON MODEL OF LOGISTIC REGRESSION

S.V. Shklyar

A logistic errors-in-variables regression is considered. The response Y_n is a binary random variable and its distribution depends on true regressor $X_n^{\rm true}$. The regression function is $\mathsf{P}[Y_n=1 \mid X_n^{\rm true}] = \left(1+e^{-\beta_0-\beta_1^\top X_n^{\rm true}}\right)^{-1}$.

Consider Berkson errors-in-variables model. Assume that the error in the regressor is homoscedastic and normally distributed. In the model, the true regressor is a hidden random variable, and $X_n^{\rm true}|X_n^{\rm obs} \sim N(X_0^{\rm obs}, T)$, where $X_0^{\rm obs}$ is an assigned/observed surrogate for $X_n^{\rm true}$. We consider both the functional model ($X_0^{\rm true}$ are non-random) and the structural model ($X_0^{\rm true}$ are independent and identically distributed).

Consider the scalar regression (the regressors $X_n^{\rm obs}$ and $X_n^{\rm true}$ are scalar variables) and assume that the parameter $T=\tau^2$ is known. If in the functional model not all observations $X_n^{\rm obs}$ are equal, then the regression parameter $\beta=(\beta_0,\beta_1)$ is identifiable. If in the structural model the distribution of $X^{\rm obs}$ is not degenerate, then β is identifiable. The result is presented in [2].

Consider the scalar regression with unknown parameter $T=\tau^2$ is known. If in the functional model $X_n^{\rm obs}$ attain at least 4 different values, then β and $\beta_1^2\tau^2$ are identifiable. If in the structural model the distribution of X_{0n} is not concentrated at 3 points, then β and $\beta_1^2\tau^2$ are identifiable.

If in the structural multiple-regression model the distribution of $X^{\rm obs}$ has non-zero absolutely continuous part, then β and $\beta_1^\top T\beta_1$ are identifiable. The situation is different from the case of classical errors [1].

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Taras Shevchenko National University, Kiev, Ukraine

E-mail address: shklyar@mail.univ.kiev.ua

ASYMPTOTIC PROPERTIES OF SIMEX ESTIMATOR IN MISCLASSIFICATION MODELS

I.A. Sivak

Consider general regression problem with response Y and random discrete regressor X that has possible outcomes $1,2,...,m$. Let X be true value that can be misclassified, and X^* be observed regressor. Misclassification error is described by a given misclassification matrix Π. The parameter of interest is β. We investigate estimators constructed by MC-SIMEX method in [1] with polynomial extrapolation function and obtain expansions of naive estimate $\widehat{\beta}_{naive} = \beta^*(\Pi)+o(1)$ a.s. as $n\to\infty$, where

$$\beta^*(\Pi) = \beta_0 + \sum_{k=1}^{l} \frac{d^k\beta(E;\Pi-E)}{k!} + O(\|\Pi-E\|^{l+1}),\ \Pi\to E \tag{1}$$

and MC-SIMEX estimate satisfies $\widehat{\beta}_{MC-SIMEX} = \beta^*_{MC-SIMEX} + o(1)$ a.s. as $n\to\infty$, where

$$\beta^*_{MC-SIMEX} = \beta_0 + O(\|\Pi-E\|^{l+1}),\ \Pi\to E \tag{2}$$

and l is degree of the polynomial extrapolation function.

Here the differential $d^k\beta(E;\Pi-E)$ denotes the value of multilinear form generated by the derivative at point E with arguments $\Pi-E$, and E is identity matrix.

It follows from (1) and (2) that MC-SIMEX estimator is closer to the true value than naive estimator as $\Pi\to E$. Relations (1) and (2) explain why MC-SIMEX approach gives better results than some consistent estimators in small and medium samples.

The results are joint with Prof. A.G. Kukush.

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National Taras Shevchenko University, Kyiv, Ukraine
E-mail address: sivirinaa@gmail.com

CHEMICAL AND FORENSIC ANALYSIS OF JFK ASSASSINATION BULLET LOTS: IS A SECOND SHOOTER POSSIBLE?

Cliff Spiegelman, Bill Tobin, Dennis James, Simon Sheather, Stu Wexler, Max Roundhill

The assassination of JFK traumatized the nation. In the talk, we show that evidence used to rule out a second assassin is fundamentally flawed. This talk discusses new compositional analyses of bullets reportedly to have been derived from the same batch as those used in the assassination. Our analyses show that the bullet fragments involved in the assassination are not nearly as rare as previously reported. In particular, our test results are compared to key bullet composition testimony presented before the House Select Committee on Assassinations (HCSA). Matches of bullets within the same box of bullets are shown to be much more likely than indicated in the HSCA' testimony. Additionally, we show that one of the ten test bullets is considered a match to one or more assassination fragments. This finding means that the bullet fragments from the assassination that match could have come from three or more separate bullets. Finally, we present a case for reanalyzing the assassination bullet fragments and conducting the necessary supporting scientific studies. These analyses will shed light on whether the five bullet fragments constitute three or more separate bullets. If the assassination fragments are derived from three or more separate bullets then a second assassin is likely, as the additional bullet would not be attributable to the main suspect, Mr. Oswald.

Texas A&M University, Statistics, 3143 TAMU, College Station, TX 77843-3143
E-mail address: cliff@stat.tamu.edu

ON THE REACTION TIME OF MOVING SUM DETECTORS

J.G. Steinebach

In this talk, we discuss some asymptotics, under the null hypothesis as well as under alternatives, concerning the reaction time of on-line monitoring schemes for detecting a "change in the mean". The stopping rules are based on "moving sums", that is, they sequentially compare a "training sample" of size m to the average of the $h = h(m)$ most recent observations. Perhaps surprisingly, the limit distributions (as $m \to \infty$) crucially depend on the asymptotic relation of h and m, posing potential problems in applications. A small simulation study will also be presented to illustrate the finite sample performance of the procedures.

The results discussed have been obtained in our joint work [1] with A. Aue (Davis), L. Horváth (Salt Lake City), and M. Kühn (Cologne).

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Mathematical Institute, University of Cologne, Weyertal 86-90, 50 931 Köln, Germany
E-mail address: jost@math.uni-koeln.de

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STATISTICAL MIXTURE ANALYIS WITH APPLICATIONS TO GENETIC DATA

O. Sugakova

Consider a sociologic study in which one is interested in connections between some individual socio-psychologic feature ξ of a person and the electoral behavior of this person at some parliament or presidential elections. Here ξ may be, e.g. "level of personal income satisfaction" or "preferable number of children in a family". The values of ξ may be derived for a sample of voters by a sociologic survey. But personal electoral preferences are "sensitive questions" which the respondents shouldn't be asked to avoid uncorrect answers. Only an official averaged information on voting results by precincts is usually available. So, merging the survey and voting information we obtain the data ξ_j, $p_{1,j}$,$\dots$,$p_{M,j}$, $j = 1, \dots, n$, where ξ_j is the value of ξ for the j-th respondent, $p_{m,j}$ is the proportion of the m-th voting behavior adherents at the precinct where the j-th respondent should vote.

Then the distribution of ξ_j is described by the model of mixture with varying concentrations

$$\mathsf{P}\{\xi_j \in A\} = p_j^1 F_1(A) + p_j^2 F_2(A) \cdots + p_j^M F_M(A),$$

where F_m is the (unknown) distribution of ξ in the subpopulation of the m-th voting behavior adherents. We will discuss the estimation of functional moments of the form $\bar{g}_m = \int g(x) F_m(dx)$ where g is some fixed function. The simplest estimate to $\bar{g}_m$ is $\hat{g}_{m,n} = \frac{1}{n} \sum a_{j,n}^m g(\xi_j)$, where $a_{j,n}^k$ are the minimax weights (see [1]). Some generalizations of this approach will be considered. Testing of hypotheses of the form $\bar{g}_m = \bar{g}_k$ will be also discussed.

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Kyiv National University, Ukraine

E-mail address: sugak@univ.kiev.ua

NON-ASYMPTOTIC CONFIDENCE INTERVALS FOR BAXTER ESTIMATES OF THE PARAMETER OF RANDOM FUNCTIONS

O.O. Synyavska

Let $\{\xi(t),\, t \in [0,1]\}$ be a stochastic process with K_1-increments [1] with zero mean and covariance function $r(t,s) = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$, where $H \in (0,1)$. For the observation of a stochastic process $\{\xi(t),\, t \in [0,1]\}$ at the points $\{\frac{k}{2^n}\big|0 \leqslant k \leqslant 2^n,\, n \geqslant 1\}$ we obtain an estimate of unknown parameters H and construct non-asymptotic confidence intervals.

Consider the sequences of Baxter sums:
$\hat{S}_n^{(1)} = 2^{n(2H-1)} \sum_{k=0}^{2^n-1} \left(\xi\left(\frac{k+1}{2^n}\right) - \xi\left(\frac{k}{2^n}\right)\right)^2$, $\hat{S}_n^{(2)} = 2^{n(2H-1)} \sum_{k=0}^{2^n-1} \left(\xi\left(\frac{k+1}{2^n}\right) - 2\xi\left(\frac{k}{2^n} + \frac{1}{2^{n+1}}\right) + \xi\left(\frac{k}{2^n}\right)\right)^2, n \geqslant 1$. Let $\theta(H) = 2^{2-2H} - 1$, $H \in [0,1]$, where $H = H(\theta)$, $\theta \in (0,3)$ is the inverse function of $\theta(H)$.

Theorem 1. *Statistics $H_n = H(\theta_n)$, $n \geqslant 1$, where $\theta_n = \hat{S}_n^{(2)}/\hat{S}_n^{(1)} \to \theta(H)$, is strongly consistent estimator of the parameter H.*

Theorem 2. *Let $H_1, H_2 \in [0,1]$ — fixed, $H_1 < H_2$, $H \in (H_1, H_2)$. Then the interval $\left(H\left(\theta_n + m_\epsilon(n)\right), H\left(\theta_n - m_\varepsilon(n)\right)\right) \cap (H_1, H_2)$, where*

$$m_\epsilon(n) \geqslant \sup_{H \in (H_1,H_2)} \frac{8\theta a_{1,n} + 2\sqrt{16\theta^2 a_{1,n}^2 + 2\left(2^n\varepsilon - 8a_{1,n}\right)\left(\theta^2 a_{1,n} + a_{2,n}\right)}}{2^n\varepsilon - 8a_{1,n}},$$

$$a_{1,n} = \sum_{l=1}^{2^n-1} \left(\frac{1}{2}(l+1)^{2H} - l^{2H} + \frac{1}{2}(l-1)^{2H}\right)^2,$$

$$a_{2,n} = \sum_{l=1}^{2^n-1} \left(-3l^{2H} + 2\left(l+\frac{1}{2}\right)^{2H} - \frac{1}{2}(l+1)^{2H} - \frac{1}{2}(l-1)^{2H} + 2\left(l-\frac{1}{2}\right)^{2H}\right)^2,$$

is a confidence interval with confidence level $1-\varepsilon$.

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Kyiv National Taras Shevchenko University, Mechanics and Mathematics Faculty, 01033 Kyiv.

E-mail address: olja_sunjavska@ua.fm

STATISTICS OF PARTIALLY OBSERVED LINEAR SYSTEMS

V. Zaiats

Our objective is to focus on a model related to partially observed linear systems, where the function we would like to control is not observed directly, and to perform estimation of different functional characteristics in this model.

Assume that we observe a process $X = (X_t,\ 0 \le t \le T)$ satisfying the following system of stochastic differential equations:

$$\begin{aligned} \mathrm{d}X_t &= h_t Y_t \,\mathrm{d}t + \varepsilon \,\mathrm{d}W_t, \quad X_0 = 0, \\ \mathrm{d}Y_t &= g_t Y_t \,\mathrm{d}t + \varepsilon \,\mathrm{d}V_t, \quad Y_0 = y_0 \neq 0, \quad 0 \le t \le T, \end{aligned}$$

where W_t and V_t, $0 \le t \le T$, are two independent Wiener processes. The process $Y = (Y_t,\ 0 \le t \le T)$ **cannot be observed** directly, but it is the one *we would like to control*.

In this model, we consider the problem of estimation of different functions on $0 \le t \le T$, in the asymptotics of a *small noise*, i.e., as $\varepsilon \to 0$. We propose some kernel-type estimators for the functions $f_t := h_t y_t$, h_t, y_t, g_t, $0 \le t \le T$, and study their properties. Here y_t, $0 \le t \le T$, is the solution of the above model with the noise dropped.

This is a joint work with Yu. Kutoyants (Université du Maine, France).

Escola Politècnica Superior, Universitat de Vic, c/. de la Laura, 13, 08500 Vic (Barcelona) Spain

E-mail address: vladimir.zaiats@uvic.es

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THE BEST CRITERIA OF CHECKING HYPOTHESES

Zurab Zerakidze

The necessary and sufficient conditions for existence of consistent criteria are obtained.

Let H be sets hypotheses and $\beta(H)$ be σ-algebra that contains all finite subsets of H.

Definition. The family of probability measures $\{\mu_h, h \in H\}$ is said to admit a consistent criterion of hypothesis if there exists even though one measurable map δ of the space (E, S) in $(H, \beta(H))$ such that $\mu_h(x : \delta(x) = h) = 1$, $\forall h \in H$. We prove the following theorems:

Theorem 1. Let $H = \{H_i, i \in N\}$. The family of probability measures $\{\mu_{H_i}, i \in N\}$ admits a consistent criterion of hypothesis if and only if the family of probability measures $\{\mu_{H_i}, i \in N\}$, $N = \{1, 2, \dots, n, \dots\}$ is strongly separable.

Theorem 2. Let $H = \{H_i, i \in N\}$ and the family of probability measures $\{\mu_{H_i}, i \in N\}$ be separable, then the family of probability measures admits a consistent criterion of hypothesis.

Theorem 3. Let $H = \{H_i, i \in N\}$ and the family of probability measures $\{\mu_{H_i}, i \in N\}$ be weakly separable, then the family of probability measures admits a consistent criterion of hypothesis.

Theorem 4. Let $H = \{H_i, i \in N\}$ and the family of probability measures $\{\mu_{H_i}, i \in N\}$ be singular then the family of probability measures admits a consistent criterion of hypothesis.

Theorem 5. On any set R with continuum power one can define strongly separable family of probability measures $\{\mu_{H_i}, i \in N\}$ that admits a consistent criterion of hypothesis.

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Gori university, Department of education, exact and natural sciences, 53 Chavchavadze str., Tbilisi, Georgia

E-mail address: z.zerakidze@mail.ru

ON LOCAL LINEAR RANK REGRESSION

Silvelyn Zwanzig

In a nonparametric regression model

$$y_i = f(x_i) + \varepsilon_i$$

with symmetric distributed independent errors ε_i a local linear rank estimator is defined as follows

$$\widehat{f}_{rank}(x) = \overline{y}_w + (x - \overline{x}_w)\widehat{\beta}_{rank}, \quad \widehat{\beta}_{rank} \in \arg\min_{\beta} D_w(\beta),$$

where $D_w(\beta)$ is the Jaeckel's dispersion

$$D_w(\beta) = \sum_{i=1}^{n} w_i\,(x)\,(y_i - \overline{y}_w - (x_i - \overline{x}_w)\beta) a_n(R_i(\beta))$$

with $\sum_{i=1}^{n} w_i\,(x) = 1$, $\overline{y}_w = \sum_{i=1}^{n} w_i\,(x)\,y_i$, $\overline{x}_w = \sum_{i=1}^{n} w_i\,(x)\,y_i$. It is related to the simple local linear regression model with eliminated intercept and heteroscedastic errors

$$w_i\,(x)\,(y_i - \overline{y}_w) = \beta w_i\,(x)\,(x_i - \overline{x}_w) + w_i\,(x)\,\varepsilon_i.$$

The $R_i(\beta)$ denotes the rank of the residuals $w_i\,(x)\,(y_i - \overline{y}_w - (x_i - \overline{x}_w)\beta)$, the scores $a_n(i)$ are generated by a score function φ. Using the results of Kuljus, Zwanzig (2012) on rank estimation with non i.i.d. errors conditions for the consistency of $\widehat{f}_{rank}(x)$ are derived. Basing on results in Zwanzig(2007), Zwanzig (2012) the application of local linear rank method to a nonparametric error-in-variables model is considered too.

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Uppsala University, Dep. of Mathematics, Box 480, 75106 Uppsala, Sweden

E-mail address: zwanzig@math.uu.se

ON THE MARTINGALE PROPERTY OF STOCHASTIC EXPONENTIALS FOR CONTINUOUS LOCAL MARTINGALES: A NEW APPROACH

H.-J. Engelbert

In the present talk, stochastic exponentials $(Z, \mathbb{F})$ of continuous local martingales $(X, \mathbb{F})$ defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ will be considered. More precisely, given a continuous local martingale $(X, \mathbb{F})$, the process $(Z, \mathbb{F})$ is defined by $Z := \mathcal{E}(X) = \exp(X - \frac{1}{2}\langle X\rangle)$, where $\langle X\rangle$ denotes the increasing process associated with X, i.e., the unique continuous increasing process such that $\langle X\rangle_0 = 0$ and $(X^2 - \langle X\rangle, \mathbb{F})$ is a local martingale. It is a simple application of Itô's formula to conclude that $(Z, \mathbb{F})$ is again a continuous local martingale. However, for many problems in the theory of stochastic processes and its applications it is of great importance to know effective criteria ensuring that $(Z, \mathbb{F})$ is not only a local martingale but a *martingale* or even a *uniformly integrable martingale*. The aim of the present talk is to give *necessary and sufficient* conditions in terms of the associated increasing process $A := \langle X\rangle$ and in terms of another but intrinsic probability measure $\mathbf{Q}$ which is locally equivalent to $\mathbf{P}$. For this purpose, a certain canonical setting for continuous local martingales will be introduced. It will be demonstrated that Novikov's and Kazamaki's conditions are simple consequences of ours. The results and the verifiability of the conditions will be illustrated by further examples for continuous local martingales $(X, \mathbb{F})$: Solutions of one-dimensional SDEs without drift, strong Markov continuous local martingales, stochastic integrals associated with certain one-dimensional diffusions. As a result, purely analytical criteria for the 'model characteristics' are obtained to ensure the martingale property of $(Z, \mathbb{F})$. In particular, if these criteria are not satisfied, then the corresponding stochastic exponential is a *strict* local martingale which is also referred to as a *bubble* in the mathematical finance literature.

Friedrich-Schiller-University Jena, Institute of Stochastics, Ernst-Abbe-Place 2–4, 07743 Jena, Germany
E-mail address: hans-juergen.engelbert@uni-jena.de

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SDE WITH SOBOLEV COEFFICIENTS

Shizan Fang

In this talk, we will present a survey concerning recent progresses on stochastic differential equations with Sobolev coefficients, based on joint works with D. Luo, H. Lee and A. Thalmaier.

I.M.B, BP 47870, Université de Bourgogne, Dijon, France
E-mail address: fang@u-bourgogne.fr

A FOURIER APPROACH OF PATHWISE INTEGRATION

P. Imkeller, N. Perkowski

In 1961, Ciesielski established a remarkable isomorphism of spaces of Hölder continuous functions and Banach spaces of real valued sequences. This isomorphism leads to wavelet decompositions of Gaussian processes giving access for instance to a precise study of their large deviations, as shown by Baldi and Roynette. We will use Schauder representations for a pathwise approach of Young integrals, employing Ciesielski's isomorphism.

Institut für Mathematik, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099, Berlin
E-mail address: imkeller@math.hu-berlin.de, perkowsk@math.hu-berlin.de

The second author is supported by Berlin Mathematical School.

ON EXTENDED STOCHASTIC INTEGRALS WITH RESPECT TO LÉVY PROCESSES

N.A. Kachanovsky

There are different generalizations of the chaotic representation property (CRP) for Lévy processes. In particular, under the Itô's approach [1] one decomposes a Lévy process L in a sum of a Gaussian process and a stochastic integral with respect to a compensated Poisson random measure, and then uses the CRP for both terms in order to construct a generalized CRP for L. The Nualart–Schoutens's approach [2, 3] consists in decomposition of a square integrable random variable in a series of repeated stochastic integrals from nonrandom functions with respect to so-called orthogonalized centered power jump processes, these processes are constructed with using of a cádlág version of L. The Lytvynov's approach [4] is based on orthogonalization of continuous monomials in the space of square integrable random variables.

In this lecture we construct the extended stochastic integral with respect to a Lévy process and the Hida stochastic derivative in terms of the Lytvynov's generalization of the CRP; establish some properties of these operators; and show that the extended stochastic integrals, constructed with use of the above-mentioned generalizations of the CRP, coincide.

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Institute of Mathematics of NASU, 3, Tereschenkivska street, Kyiv, 01601, Ukraine

E-mail address: nick2@zeos.net

MEASURABILITY OF THE STOCHASTIC INTEGRAL WITH RESPECT TO STRONG MARTINGALE, DEPENDENT ON THE PARAMETER

N.A. Kolodii

Suppose that $(\Theta,\mathcal{U})$ is a arbitrary measurable space, $(\Omega,\mathbb{F},\mathsf{P})$ is a complete probability space, $\mathbb{F}=(\mathcal{F}_z,\ z\in\mathsf{R}^2_+)$ is a two-parameter family of σ-algebras satisfying the following conditions: 1) if $x\leqslant z$, then $\mathcal{F}_x\subset\mathcal{F}_z\subset\mathcal{F}$; 2) $\mathcal{F}_0$ contains all elements $\mathcal{F}$ of zero probability; 3) $\mathcal{F}_z=\bigcap_{x>z}\mathcal{F}_x$ for any z; 4) for all x and z, σ-algebras $\mathcal{F}_x$ and $\mathcal{F}_z$ are conditionally independent with respect $\mathcal{F}_{x\wedge z}$.

Further, D denotes the space of all functions $g:\mathsf{R}^2_+\to\mathsf{R}$, such that: g is continuous on the right; for each $z\in\mathsf{R}^2_+\backslash(\mathsf{R}_+\times\{0\})$ the limit $\lim_{x\to z,z_1\leqslant x_1,x_2<z_2}g(x)$ exists; for each $z\in\mathsf{R}^2_+\backslash(\{0\}\times\mathsf{R}_+)$ the limit $\lim_{x\to z,x_1<z_1,z_2\leqslant x_2}g(x)$ exists; for each $z>0$ the limit $\lim_{z>x\to z}g(x)$ exists. For $g,g',g''\in\mathsf{D}$, we define

$$\|g\|_z=\sup_{x\in[0,z]}|g(x)|,\quad \rho(g',g'')=\sum_{k=1}^{\infty}2^{-k}(1\wedge\|g'-g''\|_{(k,k)}).$$

Further, D_0 denotes the space of functions $g\in\mathsf{D}$, such that $g(x_1,0)=g(0,x_2)=0$ for any x. $\mathcal{T}$ and $\mathcal{P}$ denote the σ-algebras of $\mathbb{F}$-progressively measurable and $\mathbb{F}$-predictable subsets $\mathsf{R}^2_+\times\Omega$ (see [1]), and $\mathcal{M}^2_S=\mathcal{M}^2_S(\mathbb{F},\mathsf{P})$ denotes the space of square-integrable strong $\mathbb{F}$-martingales.

We obtain sufficient conditions of measurability with respect to a parameter of the stochastic integral $\int_{]0,x]}\beta(\theta,u)\mu(\theta,du)$ from predictable field $\beta(\theta,\cdot)$ with respect to the two-parameter strong martingale $\mu(\theta,\cdot)$. Several results about measurability with respect to a parameter of the stochastic integral with respect to the two-parameter strong martingale are contained in [2].

$\widetilde{\mathcal{T}}$ denote the σ-algebras of subsets C of the space $\Theta\times\mathsf{R}^2_+\times\Omega$ such that $C\cap(\Theta\times[0,z]\times\Omega)\in\mathcal{U}\otimes\mathcal{B}([0,z])\otimes\mathcal{F}_z$ for any $z\in\mathsf{R}^2_+$.

Theorem 1. *Suppose that the following conditions hold:* $(\theta,(x,\omega))\mapsto\beta(\theta,x,\omega)\in\mathcal{U}\otimes\mathcal{P}|\mathcal{B}(\mathsf{R})$, $(\theta,x,\omega)\mapsto\mu(\theta,x,\omega)\in\widetilde{\mathcal{T}}|\mathcal{B}(\mathsf{R})$, $\mu(\theta,\cdot)\in\mathcal{M}^2_S$, $\mathsf{E}\int_{]0,x]}\beta^2(\theta,u)\overline{\mu}(\theta,du)<\infty$ *for any* $\theta\in\Theta,x\in\mathsf{R}^2_+$. *Then there exists a function* $\Psi(\theta,x,\omega):\Theta\times\mathsf{R}^2_+\times\Omega\mapsto\mathsf{R}$ *such that:* 1) $(\theta,x,\omega)\mapsto\Psi(\theta,x,\omega)\in\widetilde{\mathcal{T}}|\mathcal{B}(\mathsf{R})$, 2) $\Psi(\theta,x)$ *is modification of a stochastic integral on the rectangle* $]0,x]$ *of the predictable field* $\beta(\theta,\cdot)$ *with respect to the square-integrable strong martingale* $\mu(\theta,\cdot)$, 3) *for each* $\theta\in\Theta$ *the random field* $\Psi(\theta,\cdot)$ *is a element of the class* $\mathcal{M}^2_S$ *and the quadratic variation* [3] $\Psi(\theta,\cdot)$ *in the rectangle* $]0,x]$ *is equal to* $\int_{]0,x]}\beta^2(\theta,u)\,\overline{\mu}(\theta,du)$.

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Volgograd State University, Russia

E-mail address: na.kolodii@gmail.com

EVOLUTION EQUATIONS DRIVEN BY GENERAL STOCHASTIC MEASURES IN HILBERT SPACES

V.M. Radchenko

Let μ be a stochastic measure (in general sense) on Borel σ-algebra $\mathcal{B}(\mathbb{R})$, i. e. μ be a σ-additive function $\mu:\mathcal{B}(\mathbb{R})\to L_0$.

Lemma 1. *(*[1]*) Suppose there exists real-valued finite measure* m *on* $(\mathsf{X},\mathcal{B})$ *with the following property: if measurable function* $h:\mathsf{X}\to\mathbb{R}$ *is such that* $\int_{\mathsf{X}}h^2\,d\mathsf{m}<+\infty$ *then* h *is integrable w.r.t.* μ *on* X.

Let measurable functions $f_k:\mathsf{X}\to\mathbb{R},\ k\geqslant1$, *are such that* $\int_{\mathsf{X}}\left(\sum_{k=1}^{\infty}f_k^2\right)d\mathsf{m}<+\infty$. *Then* $\sum_{k=1}^{\infty}\left(\int_{\mathsf{X}}f_k\,d\mu\right)^2<+\infty$ *a. s.*

Let H be a Hilbert space with the orthonormal basis e_j, $j \geqslant 1$, $f : \mathsf{X} \to \mathsf{H}$ be a measurable function,

$$f(x) = \sum_{j=1}^{\infty} e_j f_j(x), \quad f_j : \mathsf{X} \to \mathbb{R}, \quad \sum_{j=1}^{\infty} f_j^2(x) = \|f(x)\|_{\mathsf{H}}^2 < +\infty.$$

If $\int_{\mathsf{X}} \|f(x)\|_{\mathsf{H}}^2 \, d\mathsf{m} < +\infty$ then, under the Lemma conditions, $\sum_{j=1}^{\infty} e_j \int_{\mathsf{X}} f_j(x) \, d\mu$ converge in H. We can take the sum as $\int_{\mathsf{X}} f \, d\mu$.

Theorem 2. *For measurable $f^{(n)} : \mathsf{X} \to \mathsf{H}$, $n \geqslant 1$, holds*

$$\int_{\mathsf{X}} \|f^{(n)}(x)\|_{\mathsf{H}}^2 \, d\mathsf{m}(x) \to 0 \Rightarrow \int_{\mathsf{X}} f^{(n)} \, d\mu \xrightarrow{\mathsf{P}} 0.$$

Using the constructed integral we can investigate the mild solutions to evolution equations driven by general stochastic measures.

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Taras Shevchenko National University of Kyiv, Department of Mathematical Analysis, 64, Volodymyrs'ka St., 01601 Kyiv, Ukraine

E-mail address: vradchenko@univ.kiev.ua

RECIPROCAL PROCESSES: A STOCHASTIC ANALYSIS APPROACH.

S. Roelly

Reciprocal processes (whose concept can be traced back to E. Schrödinger) form a class of stochastic processes that generalize the notion of Markov processes and are usually defined via a temporal Markovian field property. They are constructed as mixture of bridges. We propose in this talk a characterization of several types of reciprocal processes via duality formulae on path spaces. We will treat the case of reciprocal processes with continuous paths associated to Brownian diffusions and the case of reciprocal processes associated to pure jump processes.

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Institut für Mathematik der Universität Potsdam, Am Neuen Palais 10, 14469 Potsdam, Germany

E-mail address: roelly@math.uni-potsdam.de

PROPERTIES OF ITO-WIENER EXPANSION FOR BROWNIAN MOTION ON CARNOT GROUP

A. Rudenko

Consider following stochastic differential equation

$$dX(t) = V_1(X(t)) \circ dW_1(t) + \ldots + V_1(X(t)) \circ dW_n(t)$$

where $(W_1, \ldots, W_n)$ is a standart n-dimensional Brownian motion and $V_1, \ldots, V_n$ is a set of vector fields and $\circ$ denotes Stratonovich stochastic integration. Let $G = (\mathbb{R}^N, \oplus)$ be a Carnot group and g is a Lie algebra of left-invariant vector fields on G. Suppose that $V_1, \ldots, V_n$ is a set of linear independent elements of g generating g. Then the solution of such equation is called Brownian motion on G, which is known to describe solutions for a class of degenerate SDE satisfying Hormander condition [3].

We will show that kernels of Ito-Wiener expansion for $f(X(t))$ possess the same behaviour as the density of $X(t)$, meaning all of their derivatives have similar estimates. Our result is based on some known estimates for the density of $X(t)$ (see [1, 2]). In [4] we have obtained similar upper Gaussian bounds for non-degenerate SDE with smooth coefficients. These type of estimates can be used to study functionals of $X(t)$.

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Institute of mathematics NASU, 01601 Ukraine, Kiev-4, 3, Tereschenkivska st.
E-mail address: arooden@gmail.com

A PROBABILISTIC APPROXIMATION OF THE CAUCHY PROBLEM SOLUTION OF SOME EVOLUTION EQUATIONS

Natalya Smorodina

We construct an analogy of a probabilistic representation of the Cauchy problem solution of the equation $\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + f(x)u$, where σ is a complex number.

St.Petersburg State University, St.-Petersburg, Russian Federation
E-mail address: smorodin@ns2691.spb.edu

BARYCENTRIC CODE AND FUNCTIONAL INEQUALITIES

Taras Tymoshkevych

Let μ be a probability measure on Borel sets of space $\mathbb{R}^n$. We introduce family of sets

$$V_\mu(\alpha) = \{C_\mu(f) : f : \mathbb{R}^n \to [0;1], \int_{R^n} f(x)\mu(dx) = \alpha\}, \alpha \in (0;1]$$

where

$$C_\mu(f) = \frac{\int_{\mathbb{R}^n} x f(x)\mu(dx)}{\int_{\mathbb{R}^n} f(x)\mu(dx)}$$

is the barycenter of a function f with respect to the measure μ.

The family of sets uniquely restores the measure μ, that's why we call it the barycentric code.

We investigate the applicability of the new characteristics of in the functional measure inequalities of the logarithmic Sobolev inequality, because conditions in terms of barycentric code is condition of integral type.

For the inverse Sobolev inequality in n-dimensional Euclidean space, we have a generalization of the result of Bobkov [1]. We also have the necessary conditions for rotationally invariant measure, for direct Sobolev inequality in n-dimensional Euclidean space.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: vottak@mail.ru

THE PARABOLIC HARNACK INEQUALITY FOR NON-DEGENERATE DIFFUSIONS WITH JUMPS

S.V. Anulova

One of the important applications of the parabolic Harnack inequality is the establishing of ergodicity for the correspondent stochastic process. With this prospects we generalize the elliptic inequality proved in [1]. Additionally we relax the assumptions of [1]: the jump measure in it is absolutely continuous with respect to the Lebesgue measure. The proof exploits the basic construction of Krylov-Safonov for diffusions and its modernization for the purpose of adding jumps by R. Bass.

Let $B(x,r)$ denote $\{y \in \mathbb{R}^d : |y-x| < r\}$, Q be the cylinder $\{t \in [0,2), |x| \in B(0,1)\}$, and $P_{t,x}, (t,x) \in Q$, be a markov process, stopped at the moment of leaving Q, with the generator $\mathcal{L}f(t,x) = \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(t,x)\frac{\partial^2 f(t,x)}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(t,x)\frac{\partial f(t,x)}{\partial x_i} + \int_{\mathbb{R}^d\setminus\{0\}}[f(t,x+h) - f(t,x) - 1_{(|h|\le 1)} h \cdot \nabla f(t,x)]\mu(t,x;dh)$ with measurable coefficients. Denote by $m_{t,x}$ the exit measure for initial conditions (t,x).

ASSUMPTIONS There exist positive constants λ, K, k and β such that for all t,x: 1) $\lambda|y|^2 \le y^T a(t,x)y$, $y \in \mathbb{R}^d$; 2) $\|a(t,x)\| + |b(t,x)| + \int_{\mathbb{R}^d}(|h|^2 \wedge 1)\mu(t,x;dh) \le K$; 3) for any $r \in (0,1]$, y_1, $y_2 \in B(x,r/2) \cap B(0,1)$ and borel A with $\operatorname{dist}(x,A) \geqslant r$ holds $\mu(t,y_1;\{h : y_1 + h \in A\}) \le kr^{-\beta}\mu(t,y_2;\{h : y_2 + h \in A\})$.

Theorem 1. *There exists a positive constant $C(d,\ \lambda,\ K,\ k,\ \beta)$ s.t. for all $|x| \leqslant \frac{1}{2}$*

$$m_{0,x} \geqslant Cm_{1,0}.$$

The crucial point of the proof is the extension of Prop. 3.9 [1] to the cylinder.

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Inst. Control Sci., Russian Acad. Sci., 65 Profsoyuznaya, Moscow, Russia
E-mail address: anulovas@ipu.ru

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ON PROPERTIES OF A FLOW GENERATED BY AN SDE WITH DISCONTINUOUS DRIFT AND LEVY NOISE

O. Aryasova[1], A. Pilipenko[2]

We consider a stochastic flow generated by an SDE of the form

$$\begin{cases} d\varphi_t(x) = \alpha(\varphi_t(x))dt + d\xi(t), \\ \varphi_0(x) = x, \end{cases}$$

where $x \in \mathbb{R}$, α is a function on $\mathbb{R}$ that can be discontinuous, $(\xi(t))_{t\ge 0}$ is a Levy process.

We study the differentiability, the possibility of coalescing (meeting) and asymptotic properties of the flow.

[1] Institute of Geophysics, National Academy of Sciences of Ukraine, Palladin av. 32, 03680, Kiev-142, Ukraine
E-mail address: oaryasova@mail.ru

[2] Institute of Mathematics, National Academy of Sciences of Ukraine, Tereshchenkivska str. 3, 01601, Kiev, Ukraine
E-mail address: apilip@imath.kiev.ua

STOCHASTIC DIFFERENTIAL INCLUSIONS FOR THE STOCHASTIC PROCESSES IN TERMS OF THEIR GENERATORS

S.V. Azarina

The idea of this work is to prove the existence of stochastic process on a manifold, if the inclusion for its generator is given. This can be done using the results for the stochastic differential inclusions in terms of mean derivatives on euclidean space and manifold, that were obtain together with Yu.E. Gliklikh (e.g. [1, 2]).

Let M be a finite dimensional connected Riemannian manifold and τM be a second order tangent bundle on it (see [3]). Note that the generator of the stochastic processes of Belopolskaya-Daletskii form is the second order tangent vector field. Denote by $(\cdot,\cdot)$ metric on τM. By the square of the norm of set B from $\tau_x M$ we will understand $\sup_{b\in B}(b,b)$.

Theorem 1. *For any upper semicontinuous $\mathbf{G}(\mathbf{t},\mathbf{x})$ set-valued second order tangent vector field, $t \in [0,T]$ with closed convex images, uniformly bounded w.r.t. to the above-mentioned norm there exist a certain probability space (P) and a stochastic process on it such that its generator at each moment t is P-a.s. from $\mathbf{G}(\mathbf{t},\mathbf{x})$.*

As an example we will consider the inclusion of such type on a loop manifold.

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Voronezh State Technical University, Department of highest mathematics and of physical and mathematical modelling, Moskovsky prosp. 14, Voronezh, Russia

E-mail address: azarina@math.vsu.ru

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SPACE-TIME STATIONARY SOLUTIONS FOR THE BURGERS EQUATION WITH RANDOM FORCE

Yuri Bakhtin

The Burgers equation is one of the basic hydrodynamic models that describes the evolution of velocity fields of sticky dust particles. When supplied with random forcing it turns into an infinite-dimensional random dynamical system that has been studied since late 1990's. The variational approach to Burgers equation allows to study the system by analyzing optimal paths in the random landscape generated by random force potential. Therefore, this is essentially a random media problem. For a long time only compact cases of Burgers dynamics on the circle or a torus were understood well. However, recently a lot of progress was achieved for noncompact cases with forcing based on Poissonian point field.

The talk is naturally split into three parts. In the first part I will recall basics on the Burgers equation and explain the central ergodic result, so called One Force — One Solution (1F1S) Principle, for Burgers dynamics with random force in the compact case. The second part will be devoted to localization and 1F1S in the quasi-compact case where the random forcing decays to zero at infinity. In the third part I will talk about 1F1S for the case of the forcing that is stationary in space-time and has no decay at infinity. This part is based on techniques that have been developed for last-passage percolation and related models. It involves Kesten's concentration inequality, greedy lattice animals, and other tools of modern probability theory.

School of Mathematics, 686 Cherry St., Atlanta

E-mail address: yuri.bakhtin@gmail.com

FBSDES AND FULLY NONLINEAR SYSTEMS OF PARABOLIC EQUATIONS

Ya. Belopolskaya

Consider a backward stochastic differential equation

$$dy(\theta) = -g(w_x(\theta), y((\theta), z(\theta))d\theta + z(\theta)dw(\theta), \quad y(T) = v_0(w_x(T)) \in R^{d_1}, \tag{1}$$

where $w(\theta) \in R^d$ is a Wiener process, $w_x(\theta) = x + w(\theta) - w(t)$. It was proved in [1] that when the couple $(y(\theta) \in R^{d_1}, z(\theta) \in R^d \times R^{d_1})$ satisfies (1), then $v(t,x) = y(t)$ is a viscosity solution to a system of quasilinear PDEs of the form

$$\frac{\partial v_l}{\partial t} + \frac{1}{2}\Delta v_l + g(x, v, \nabla u) = 0, \quad v_l(T,x) = v_{l0}(x), \quad x \in R^d. \tag{2}$$

We extend this result to a class of fully nonlinear parabolic systems. Namely, we consider the Cauchy problem

$$\frac{\partial u_l}{\partial t} + F(x, u, \nabla u, \nabla^2 u_l) = 0, \quad u_l(T,x) = u_{l0}(x), \quad x \in R^d, \tag{3}$$

with a C^1-smooth function $F : R^d \times R^{d_1} \times M \times M_1 \to R$ where $M = R^d \otimes R^{d_1}, M_1 = R^d \otimes R^d \otimes R^{d_1}$.

To construct a viscosity solution to this problem we derive the first differential prolongation of (3) and consider a new system of PDEs with respect to a new function $V(t,x) = (u(t,x), \nabla u(t,x))$ having two components. This allows us to include (3) into a system of the type (2) (though much more complicate) which is nevertheless nonlinear in V and ∇V but linear w.r.t $\nabla^2 V$. Next we construct a fully coupled forward-backward stochastic differential equation (FBSDE) associated with the new system of parabolic equations and state conditions on F which are sufficient to prove the existence and uniqueness of a solution to the FBSDE. As a last step we prove that a component of the FBSDE solution gives rise to a viscosity solution of (3). The proofs of the corresponding results can be found in papers [2],[3].

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St.Petersburg State University for Architecture and Civil Engineering, 190005, St.Petersburg, 2-ja Krasnoarmejskaya 4
E-mail address: yana@yb1569.spb.edu

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STOCHASTIC CONTROL OF STOCHASTIC PROCESSES WITH LÉVY NOISE, BASED ON THE TIME-CHANGE TRANSFORMATIONS

S.V. Bodnarchuk

Consider a stochastic differential equation of the form

$$X(x,t) = x + \int_0^t a(X(x,s))\,ds + Z(t), \quad x \in \mathbb{R}^m, \quad t \in \mathbb{R}^+, \tag{1}$$

where Z is a Lévy process in $\mathbb{R}^m$ without diffusion component.

Consider the following local Doeblin condition: for every $R > 0$ there exists $T = T(R)$ such that

$$\delta(T,R) = \inf_{|x|,|y|\le R} \int_{\mathbb{R}^m} (P_x^T \wedge P_y^T)(dz) > 0,$$

where $P_x^t(\cdot) = P(X(t) \in \cdot | X(0) = x)$, and, for any two probability measures μ, κ

$$(\mu \wedge \kappa)(dz) = \min\left(\frac{d\mu}{d(\mu+\kappa)}(z), \frac{d\kappa}{d(\mu+\kappa)}(z)\right)(\mu+\kappa)(dz)$$

Local Doeblin condition is one of the sufficient conditions for the exponential ergodicity of solution of (1) (see [1]). In the report we consider the conditions which are sufficient for the local Doeblin condition.

For s fixed set Γ with $\Pi(\Gamma) < +\infty$ (Π is a measure Lévy of the process Z) and for $\rho, \varrho > 0$, $\sigma \in (0,1)$ we define

$$\Pi_1 = \inf_{x\in\mathbb{R}^m} \Pi(u \in \Gamma : |a(x+u) - a(x)| > \rho, |u| \le \varrho),$$

$$\Pi_2 = \inf_{x\in\mathbb{R}^m, l\in\mathbb{S}^m} \Pi(u \in \Gamma : |a(x+u) - a(x)| > \rho, |u| \le \varrho, a(x+u) - a(x) \in V_\sigma(l)),$$

where $V_\sigma(l)$ is a two-sided cone with the axis l and the angular coefficient σ.

Theorem 1. *Suppose that the function $a \in C^2$ has bounded first and second derivatives. If $\Pi_1 > 0$ and $\Pi_2 > 0$ then the local Doeblin condition holds true.*

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NTUU "KPI", 37 Prospect Peremogy, Kiev 03056, Ukraine
E-mail address: sem_bodn@ukr.net

ASYMPTOTIC BEHAVIOR IN MEAN SQUARE OF SOLUTIONS OF SYSTEM OF STOCHASTIC DIFFERENTIAL-DIFFERENCE EQUATIONS WITH POISSON PERTURBATIONS

N.P. Bodryck

On the probability base $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ the system of stochastic differential-difference equation [1] (SDDE) is considered

$$dx(t) + \sum_{j=0}^m A_j x(t - \triangle_j)\,dt = \sum_{j=0}^m \psi_j(\xi(\omega))\,dW_j(t)\,x(t-\Delta_j) + $$

$$+ \sum_{j=0}^m \psi_j(\xi(\omega)) \int\limits_U f_j(u) B_j x(t-\Delta_j)\,\widetilde{v}(dt,du) \tag{1}$$

with initial conditions

$$x(t)\,|_{t\in[-\triangle,0]} = \varphi(t) \in \mathbb{R}^n, \quad \Delta = \sup_j, \triangle_0, \Delta_j > 0, j = \overline{1,n}. \tag{2}$$

Theorem 1. *Let roots of characteristic quasi polynomial*

$$detV(\lambda) \equiv det\left(I \cdot \lambda + \sum_{j=0}^n A_j e^{-\Delta_j \lambda}\right)$$

are situated in left half-plane $Re\lambda < 0$.

Then: 1) if eigenvalues $\lambda(D)$ of matrix

$$D \equiv \frac{1}{\pi}\sum_{j=0}^{m}\left\{\mathbb{E}\left\{{\psi_j}^2(\xi)\right\}\left[V^{-1}(is)\right]_+^2 S_j ds + \int\limits_U \mathbb{E}\left\{\psi_j(\xi)\right\} f_j^2(u)\frac{du}{\mid u\mid^{m+1}}\int\limits_0^\infty \left[V^{-1}(is)\right]_+^2 ds\right\}$$

are satisfied the condition $\mid \lambda(D) \mid < 1$, then trivial solution of system (1), (2) is asymptotic stable in mean square; 2) if any eigenvalues $\lambda(D)$ of matrix D is satisfied the condition $\mid \lambda(D) \mid < 1$, then in arbitrary small zero neighborhood will be found initial function $\phi(t)$ such, that $\lim\limits_{t\to+\infty}\mu(t) = +\infty$.

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Chernivtsi national university named after Yurij Fedkovich, Chernivtsi, Ukraine
E-mail address: bodryk-n@yandex.ru

STOCHASTIC DIFFERENTIAL EQUATIONS ON TIME SCALES

A.O. Bratochkina, O.M. Stanzhytskyi

The theory of time scales was presented by Stefan Hilger in 1988 [1]. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$.

The construction of the stochastic integral on general time scale is based on a proved connection between Δ-Lebesgue integral and Lebesgue integral in common sense. Herewith if $\mathbb{T} = \mathbb{R}$ then the presented integral transforms in the standard Ito-type stochastic integral, if $\mathbb{T} = \mathbb{Z}$ then such integral is the stochastic integral on discrete time scale which was introduced by Suman Sanyal in his PhD dissertation [3].

For next stochastic differential equation on general time scale

$$\Delta X = b(X,t)\Delta t + B(X,t)\Delta W(t)$$
$$X(0) = X_0$$

we introduced a concept of the solution, proved an existence and uniqueness theorem, also established the Markov property of the solution.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: sestri4kina@ukr.net, stom@mail.univ.kiev.ua

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INVARIANCE, MONOTONICITY AND COMPARISON PRINCIPLE FOR STOCHASTIC DELAY DIFFERENTIAL EQUATIONS

I.D. Chueshov

We deal with invariance and monotonicity properties for the class of Kunita-type stochastic differential equations in $\mathbb{R}^d$ with delay. We present several types of sufficient conditions for the invariance of closed subsets of $\mathbb{R}^d$, establish a comparison principle and show that under appropriate conditions the stochastic delay system considered generates a monotone (order-preserving) random dynamical system. This makes it possible to apply some structural results from the theory of monotone random systems (see [3] and the references therein) to describe pullback dynamics. Several applications are considered. Some generalizations to the SPDE case is also discussed.

The talk is partially based on the result which were obtained in collaboration with M. Scheutzow (see [4, 5]) and also with Sevilla group (see [1, 2]).

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Kharkov National University, Kharkov, 61022, Ukraine
E-mail address: chueshov@univer.kharkov.ua

ON SOME PROPERTIES OF ESTIMATION FOR THE DRIFT COEFFICIENT OF A STOCHASTIC DIFFERENTIAL EQUATION ON THE PLANE WITH ADDITIVE FBM

O.M. Deriyeva, S.P. Shpyga

Real stochastic fields $\{x(z), z \geq 0\}$, $\{y(z), z \geq 0\}$ and fBm $\{B(z), z \geq 0\}$ on the plane $z = (s,t) \in \mathbb{R}^2$ are considered. The stochastic process $\{y(z), z \geq 0\}$ is assumed to possessed the stochastic differential equation

$$dy(z) = a_0(z)x(z)dz + dB(z), \quad z \geq 0.$$

The problem is to estimate drift coefficient (i.e. unknown function a_0) from the observation of $\{(x(z), y(z)), z \geq 0\}$ on $z \in [0,T]^2$. Maximum likelihood estimator is studied. Strong consistency and asymptotic normality of estimator are proved. Results are applied to the special case when $x(s,t) = 1$ under additional constraints.

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V.M.Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine
E-mail address: derieva@gmail.com, shpyga@meta.ua

EXPONENTIAL FUNCTIONALS OF BROWNIAN MOTION AND EXPLOSION TIMES OF A SYSTEM OF SEMILINEAR SPDE'S

M. Dozzi[1], E.T. Kolkovska[2], J.A. Lopez-Mimbela[3]

We investigate lower and upper bounds for the blowup times of the system of semilinear SPDE's given by

$$\begin{aligned} du_1(t,x) &= [(\Delta + V_1)u_1(t,x) + u_2^p(t,x)]dt + \kappa_1 u_1(t,x)dW_t, \\ du_2(t,x) &= [(\Delta + V_2)u_2(t,x) + u_1^q(t,x)]dt + \kappa_2 u_2(t,x)dW_t, \quad x \in D \subset \mathbb{R}^d, \end{aligned}$$

with Dirichlet boundary conditions, where D is a bounded smooth domain, $V_i > 0$, $\kappa_i \neq 0$ are positive constants, $i = 1, 2$, and W is standard Brownian motion. Under certain conditions on the system parameters we obtain explicit solutions of a related system of random PDE's, which allows us to use a formula due to M. Yor to obtain the distribution functions of several explosion times. We also give the Laplace transforms at independent exponential times of related exponential functionals of Brownian motion.

[1] IECN, Lorraine University, France
E-mail address: dozzi@iecn.u-nancy.fr
[2] CIMAT, Guanajuato, Mexico
E-mail address: todorova@cimat.mx
[3] CIMAT, Guanajuato, Mexico
E-mail address: jalfredo@cimat.mx

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STOCHASTIC EQUATIONS WITH MEAN DERIVATIVES ON FIBER BUNDLES WITH CONNECTIONS AND APPLICATIONS

Y.E. Gliklikh

On the basis of the general theory of stochastic differential equations and inclusions in terms of mean derivatives developed previously by the author (see [1]), an equation of Newton-Nelson type on the vector bundle whose right-hand side is generated by the curvature form, is introduced and investigated. We deal with two cases: where the base is a Riemannian manifold and the bundle fibers are real vector spaces and where the base is a space-time of General Relativity and the bundle fibers are complex vector spaces. Existence of solution theorems for both cases are obtained.

In the latter case of complex bundle over a space-time of General relativity, the equation is interpreted as the one describing the motion of a quantum particle in the classical gauge field in the language of Nelson's stochastic mechanics. For special cases of symmetry groups $U(1)$, $SU(2)$ and $SU(3)$ the interrelations with usual descriptions of such motion are described. Previously the case of $U(1)$ was investigated in [2].

This is a joint work with **Natalia Vinokurova**.

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Voronezh State University, Mathematics Faculty, Universitetskaya pl. 1, 394006, Voronezh, Russia
E-mail address: yeg@math.vsu.ru

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EXISTENCE AND UNIQUENESS OF A WEAK SOLUTION OF THE CAUCHY PROBLEM FOR THE HEAT EQUATION

D.M. Gorodnya

In this paper we define this solution as generalized random function (g.r.f.) defined on $D(\mathbb{R}^{d+1})$ of smooth functions with compact support. Another approach can be found in [1].

Properties of stochastic measures and integral $\int_A f d\mu$, where A is a Borel set, f a real measurable function, μ a stochastic measure are studied in [2].

Lets consider the Cauchy problem for the heat equation with stochastic measure, which in the symbolic notation has the follow form:

$$\frac{\partial V}{\partial t} = a^2 \triangle_x V + \dot{\mu},\ t > 0, \qquad V\mid_{t=0+} = \dot{\nu}, \tag{1}$$

on the unknown g.r.f. V, where $a \in \mathbb{R},\ a > 0$, μ is a stochastic measure on the Borel σ-algebra $\mathbf{B}(\mathbb{R}^{d+1})$ equal to zero on all measurable subsets of the set $K := \{(x,t) \in \mathbb{R}^{d+1}\ :\ t < 0\}$, ν is a stochastic measure on $\mathbf{B}(\mathbb{R}^d)$.

Theorem 1. *The Cauchy problem (1) has an unique solution in the set of all generalized random functions for which exists a convolution with $\mathcal{E}$. This solution is represented by follows:*

$$(V,\varphi) = \int_{\mathbb{R}^d\times[0;\infty)} d\mu(x,t) \int_t^{\infty} ds \int_{\mathbb{R}^d} \mathcal{E}(y-x, s-t)\varphi(y,s)dy +$$

$$\int_{\mathbb{R}^d} d\nu(x) \int_0^{\infty} ds \int_{\mathbb{R}^d} \mathcal{E}(y-x,s)\varphi(y,s)dy, \quad \varphi \in D(\mathbb{R}^{d+1}),$$

where $\mathcal{E}(x,t) = \frac{\theta(t)}{(2a\sqrt{\pi t})^d} e^{-\frac{|x|^2}{4a^2 t}}, \quad (x,t) \in \mathbb{R}^{d+1}$, is the fundamental solution of the heat operator.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: gorodnia.dariia@gmail.com

G-PDE WITH LINEAR UNBOUNDED FIRST ORDER TERM IN INFINITE DIMENSIONAL SPACES

Anton Ibragimov

Despite of increasing the popularity of the theory of G-expectation the most part of results are dedicated to the finite dimensional case. In this talk I would like to present some results obtaining in infinite dimensions.

Let H is a separable Hilbert space. Consider monotone, sublinear, $L(H)$-continuous functional defined on the set of linear, bounded , non-negative, symmetric operators. We will call it G-function. Every G-function can be represented in the form $G(A) = \dfrac{1}{2} \sup\limits_{B\in\Sigma} \operatorname{Tr}[A\cdot B]$. G also defines G-normal distributed random variable with covariance set Σ: $X \sim N_G(0,\Sigma)$; the sublinear expectation $\mathrm{E}[\cdot]$ which we will call G-expectation, such that $G(A) := \dfrac{1}{2}\,\mathrm{E}[\langle AX, X\rangle]$; and G-Brownian motion $B_t \sim N_G(0, t\cdot\Sigma)$.

G-Brownian motion is related to a fully nonlinear partial differential equation in the following way:

If (B_t) is a G-Brownian motion and $u(t,x) := \mathrm{E}[\phi(x + B_{T-t})]$, then u is the viscosity solution of the following G-heat equation:

$$\frac{\partial u}{\partial t}(t,x) + G\left(\frac{\partial^2 u}{\partial x^2}\right)(t,x) = 0$$
$$u(T,x) = \phi(x)$$

Together with it we can introduce the stochastic integral over G-Brownian motion. One of the essential properties of it is obtained Itô's isometry inequality:

$$\mathrm{E}\Big[\| \int_0^T \Phi(t)\, dB_t\|_{\mathsf{H}}^2\Big] \le \int_0^T \mathrm{E}\Big[\sup_{Q\in\Sigma} \operatorname{Tr}\big[\Phi Q \Phi^*\big]\Big] dt.$$

This helps us to introduce correctly Ornstein-Uhlenbeck process $I_t = \int\limits_0^t e^{(t-s)A} dB_s$, where A is generator of C_0-semigroup (e^{tA}).

It turns out that if process $X_t := e^{(T-t)A}x + \int\limits_t^T e^{(t-\sigma)A} dB_\sigma$ and $u(t,x) := \mathrm{E}[\phi(X_t)]$, then u is the viscosity solution of the following (Kolmogorov) G-PDE:

$$\frac{\partial u}{\partial t}(t,x) + G\left(\frac{\partial^2 u}{\partial x^2}\right)(t,x) + \langle Ax, \nabla_x u(t,x)\rangle = 0$$
$$u(T,x) = \phi(x)$$

So, in this talk we try to give an explicit notion of G-expectation theory in infinite dimensions and their connection to the viscosity solutions of some types of PDEs.

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Università degli Studi di Milano-Bicocca, Dipartimento di matematica e applicazioni, via R.Cozzi,53, Milano, Italy
E-mail address: ibrahimov.ag@gmail.com

THE GENERALIZED ITÔ-WENTZELL'S FORMULA FOR THE GENERALIZED ITÔ'S EQUATION

E.V. Karachanskaya

We obtained the differential for a scalar function, which is solution of the generalized Itô's equation (GSDEI) and depends on a random process, that subordinated to some GSDEI. The formula () is named *the generalized Itô-Wentzell's formula*.

$$\begin{aligned} d_t F(t;\mathbf{x}(t)) &= Q(t;\mathbf{x}(t))dt + D_k(t;\mathbf{x}(t))dw_k + b_{i,k}(t)\frac{\partial}{\partial x_i}F(t;\mathbf{x}(t))dw_k + \\ &+ \Big[a_i(t)\frac{\partial}{\partial x_i}F(t;\mathbf{x}(t)) + \frac{1}{2}b_{i,k}(t)b_{j,k}(t)\frac{\partial^2 F(t;\mathbf{x}(t))}{\partial x_i \partial x_j} + b_{i,k}(t)\frac{\partial}{\partial x_i}D_k(t;\mathbf{x}(t))\Big]dt + \\ &+ \int_{\mathbf{R}(\gamma)}\Big[F(t;\mathbf{x}(t) + g(t;\gamma)) - F(t;\mathbf{x}(t)) + G(t;\mathbf{x}(t) + g(t;\gamma);\gamma)\Big]\nu(dt;d\gamma). \end{aligned} \tag{1}$$

where $F(t;\mathbf{x}) \in \mathbf{R}$; $\mathbf{x}(t) = \mathbf{x}(t;\omega)$, $\mathbf{x}(t)$, $a(t)$, $g(t;\gamma) \in \mathbf{R}^n$; $B(t) \in \mathbf{R}^n \times \mathbf{R}^m$, $\mathbf{w}(t) \in \mathbf{R}^m$; $\nu(\Delta t; \Delta\gamma)$ – is homogeneous on t non centered Poisson measure on $[0,T] \times \mathbf{R}^{n'}$;

$$\begin{aligned} d\mathbf{x}(t) &= a(t)dt + B(t)d\mathbf{w}(t) + \textstyle\int_{\mathbf{R}(\gamma)} g(t;\gamma)\nu(dt;d\gamma), \\ d_t F(t;\mathbf{x}) &= Q(t;\mathbf{x})dt + D_k(t;\mathbf{x})dw_k(t) + \textstyle\int_{\mathbf{R}(\gamma)} G(t;\mathbf{x};\gamma)\nu(dt;d\gamma). \end{aligned} \tag{2}$$

We determined conditions for coefficients of Eq. (2), and present two methods of the formula () development [1]. The first method is based on a classic theory of the GSDEI. The second method uses the presentation for kernels of the integral invariants and the way of construction of the automorphic functions continuum [1, 3, 4].

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Russia, Khabarovsk, Pacific National University
E-mail address: EKarachanskaya@mail.khstu.ru

INFINITE-DIMENSIONAL SEMILINEAR SDE'S DRIVEN BY CYLINDRICAL LÉVY PROCESSES

A.M. Kulik

We study semilinear SDE's of the form

$$dX_t = \Big(AX_t + f(X_t)\Big)dt + dZ_t \tag{1}$$

in a real separable Hilbert space H, where A is a generator of strongly continuous semigroup of operators in H, and Z_t is a cylindrical Lévy process in H. We propose a new approach for studying basic structural properties of the solution to (1), such as existence and uniqueness, stochastic continuity, Markov property, Feller property. This approach, unlike the one developed recently (see [1] and references therein), does not involve the assumption that the cylindrical Lévy process Z_t is decomposable in a series of eigenvectors of the operator A weighted by independent one-dimensional Lévy processes. This allows us, in particular, to study SPDE's with the α-stable space-time noise corresponding to an α-stable sheet.

For the solution to the respective linear equation (i.e. (1) with $f \equiv 0$), an important feature, revealed in [2], is that the trajectories of the solution may be time-irregular; that is, do not belong to $\mathbb{D}(\mathbb{R}^+, H)$. We introduce a proper regularization procedure, which makes it possible to solve (1) with unbounded non-linearities f, and moreover to consider equations of a form more general than (1) with non-linearities involved into the stochastic integral part.

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Institute of Mathematics, Ukrainian National Academy of Sciences, 01601 Tereshchenkivska str. 3, Kyiv, Ukraine
E-mail address: kulik@imath.kiev.ua

COMPARISON THEOREMS FOR SOME TYPES OF SOLUTIONS OF THE PARABOLIC SPDE

S.A. Melnik

On a stochastic basis $\left(\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t\geq 0}, \mathbb{P}\right)$ we consider such Couchy problem

$$du^{(j)}(t,x) = \Delta a(t,x,u^{(j)}(t,x))dt + b^{(j)}(t,x,u^{(j)}(t,x))dt + c(t,x,u^{(j)}(t,x))dw(t),$$
$$t \geq 0,\ x \in \mathbf{R}^n,\ u^{(j)}(0,x) = u_0^{(j)}(x) \geq 0,\ j = 1,2,$$

with a triple of functional spaces $\mathbf{V} \subset \mathbf{H} \equiv \mathbf{H}^* \subset \mathbf{V}^*$.
Denote $\mathbf{L} = \mathbf{C}\left([0;T]; \mathbf{L}_2(\mathbf{H} \times \Omega)\right) \cap \mathbf{L}_p([0;T] \times \Omega; \mathbf{V}),\ p \geq 2$.

Theorem 1. *Let following conditions are satisfied.*
1. $u_0^{(1)}(x) \leq u_0^{(2)}(x), \quad \forall x \in \mathbf{R}^n$.
2. $\exists\, \mathcal{K}(\mathcal{N}) > 0 : |b_u^{(j)}(t,x,u)| \leq \mathcal{K}(\mathcal{N}),\ j = 1,2, \quad |c_u(t,x,u)| \leq \mathcal{K}(\mathcal{N}),\ if\ |u| \leq \mathcal{N}$.
3. Almost all paths of processes $u^{(1)}(t,x)$ *and* $u^{(2)}(t,x)$ *are the continuous bounded functions at* $(t,x) \in [0;T] \times \mathbf{R}^n$.
Then $u^{(1)}(t,x) \leq u^{(2)}(t,x)$ *for all* $(t,x) \in [0;T] \times \mathbf{R}^n$ *with probability 1.*

Theorem 2. *There exists such* $\lambda > 0$ *that following conditions are satisfied.*
1. $u_0^{(1)}(x) \leq u_0^{(2)}(x), \quad \forall x \in \mathbf{R}^n$.
2. $b^{(2)}(t,x,u^{(2)}) \geq b^{(1)}(t,x,u^{(1)}),\ \forall (t,x) \in [0;T] \times \mathbf{R}^n,\ u^{(2)} \leq u^{(1)}$.
3. $\mathbb{E}\left(\int\limits_0^T \left(\int |c(t,x,u(t,x))| e^{-\lambda|x|} dx\right)^2 ds\right)^{0.5} < +\infty,\ \forall u \in \mathbf{L}$.
4. Function $|c_u(t,x,u)|$*, for all* $(t,x) \in [0;T] \times \mathbf{R}^n$*, either is convex downwards up to* u*, or is a constant.*
5. $\mathbb{E}\int\limits_0^T \int |c_u(t,x,u(t,x))|^2 e^{-\lambda|x|} dx ds < +\infty,\ \forall u \in \mathbf{L}$.
6. $\sup\limits_{0\leq t\leq T} \mathbb{E}\int |u^{(j)}(t,x))| e^{-\lambda|x|} dx < +\infty,\ j = 1,2$.
Then $u^{(1)}(t,x) \leq u^{(2)}(t,x)$ *for all* $(t,x) \in [0;T] \times \mathbf{R}^n$ *with probability 1.*

Institute of applied mathematics and mechanics of NASU, Rose Luxemburg str. 74, Donetsk, 83114, Ukraine
E-mail address: melnik@iamm.ac.donetsk.ua

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SINGULAR CONTROL AND OPTIMAL STOPPING OF SPDES, AND BACKWARD SPDES WITH REFLECTION

Bernt Øksendal

In Section 2 we consider general singular control problems for random fields given by a stochastic partial differential equation (SPDE). We show that under some conditions the optimal singular control can be identified with the solution of a coupled system of SPDE and a reflected backward SPDE (RBSPDE). As an illustration we apply the result to a singular optimal harvesting problem from a population whose density is modeled as a stochastic reaction-diffusion equation. In Section 3, existence and uniqueness of solutions of RBSPDEs are established. In Section 4 we prove a relation between RBSPDEs and optimal stopping of SPDEs, and in Section 5 we apply the result to a risk minimizing optimal stopping problem.

The presentation is based on joint work with Agnès Sulem, INRIA-Paris/Roquencourt and Tusheng Zhang, University of Manchester, UK.

University of Oslo, P.O. Box 1072, Blindern, 0316, Oslo, Norway, The Faculty of Mathematics and Natural Sciences Department of Mathematics

E-mail address: oksendal@math.uio.no

MIXED STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN BY WIENER PROCESS AND ALMOST SURELY HÖLDER CONTINUOUS PROCESS WITH HÖLDER EXPONENT $\gamma > 1/2$

S.V. Posashkova

We consider stochastic differential equations involving possibly dependent Wiener process and almost surely Hölder continuous process with Hölder exponent $\gamma > 1/2$, with nonhomogeneous coefficients and random initial conditions which depend on a parameter.On the one hand, such models include Wiener process that represents randomness in the sense of the memory lack. On the other hand, most of long-ranged-dependent processes have Hölder continuous trajectories with exponent greater than $1/2$. The processes in hydrodynamics, telecommunications, economics, finances demonstrate availability of random noise that can be modeled by Wiener process and also the so-called long memory that can be modeled with the help of, for example, fractional Brownian motion with Hurst index $H > 1/2$.

The assumptions on coefficients and initial conditions supplying continuous dependence of the solution on a parameter, with respect to the Besov space norm, are established.

Taras Shevchenko National University of Kyiv, The Faculty of Mechanics and Mathematics, Department of Probability Theory, Statistics and Actuarial Mathematics, Volodymirska 64, Kyiv, Ukraine

E-mail address: svposashkova@gmail.com

RANDOMIZED EULER ALGORITHM FOR THE APPROXIMATION OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH TIME–IRREGULAR COEFFICIENTS

Paweł Przybyłowicz, Paweł Morkisz

We consider pointwise approximation of solutions of scalar stochastic differential equations (SDEs) of the form:

$$\begin{cases} dX(t) = a(t, X(t))dt + b(t)dW(t), \quad t \in [0, T], \\ X(0) = \eta, \end{cases} \tag{1}$$

where the initial–value η is independent of the driving one–dimensional Brownian motion W and $\mathbb{E}|\eta|^q < +\infty$ for all $q \geq 1$. We assume that a coefficient $a : [0, T] \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function i.e. for all $y \in \mathbb{R}$, $a(\cdot, y) : [0, T] \to \mathbb{R}$ is bounded, measurable and for all $t \in [0, T]$, $a(t, \cdot) : \mathbb{R} \to \mathbb{R}$ is globally Lipschitz continuous in $\mathbb{R}$. For a coefficient $b : [0, T] \to \mathbb{R}$ we assume that it is piecewise Hölder continuous in $[0, T]$ with the exponent $\varrho \in (0, 1]$.

It is well-–known that under such assumptions on a we have the lack of convergence of the classical Euler algorithm in the worst case model of error, as long as we sample the coefficient a with respect to the time variable t only at points chosen in deterministic way. This obstacle can be overcome by considering Monte Carlo algorithms. We construct a randomized Euler algorithm X^{RE}, which evaluates function a at randomly chosen points.

The error of the algorithm X^{RE} is measured as a distance, computed in L^q norm with $q \geq 1$, between real value of $X(T)$ and the output of the algorithm. We show that the error of the new algorithm is $O(n^{-\min\{1/2, \varrho\}})$, where n is the number of discretization points. We also show that this bound is sharp.

Finally, we show that numerical examples confirm obtained theoretical results.

University of Science and Technology, Faculty of Applied Mathematics, Frydek, ul. Sportowa 4, 43-227 Miedzna, Krakow

E-mail address: pprzybyl@agh.edu.pl, morkiszp@student.agh.edu.pl

FORWARD-BACKWARD SDES DRIVEN BY LÉVY PROCESSES AND APPLICATION TO OPTION PRICING

Evelina Shamarova, Rui Sá Pereira

Recent developments of financial markets have revealed the limits of Brownian motion pricing models when they are applied to actual markets. Lévy processes, that admit jumps over time, have been found more useful for applications. Thus, we suggest a Lévy model based on Forward-Backward Stochastic Differential Equations (FBSDEs) for option pricing in a Lévy-type market. We show the existence and uniqueness of a solution to FBSDEs driven by a Lévy process L_t. Specifically, we are concerned with the following FBSDEs:

$$\begin{cases} X_t = x + \int_0^t f(s, X_s, Y_s, Z_s)\, ds + \sum_{i=1}^{\infty} \int_0^t \sigma_i(s, X_{s-}, Y_{s-})\, dH_s^{(i)}, \\ Y_t = h(X_T) + \int_t^T g(s, X_s, Y_s, Z_s)\, ds - \sum_{i=1}^{\infty} \int_t^T Z_s^i\, dH_s^{(i)}, \end{cases} \tag{1}$$

where $H^{(i)}$'s are the orthogonalized Teugels martingales associated to L_t. The results of this work are important from the mathematical point of view, and also, provide a much more realistic approach to option pricing compared to Brownian motion market models. Moreover, we discuss the existence of replicating portfolios and present an analog of the Black–Scholes PDE in a Lévy-type market.

Faculdade da Ciências, Universidade do Porto, Departamento de Matemática, Rua de Campo Alegre 687
E-mail address: evelinas@fc.up.pt, manuelsapereira@gmail.com

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JJUSTIFICATION OF THE FOURIER METHOD FOR EQUATIONS OF RECTANGULAR MEMBRANE VIBRATIONS WITH RANDOM INITIAL CONDITIONS

A.I. Slyvka-Tylyshchak

Consider boundary-value problem for a hyperbolic equation of rectangular membrane's vibrations [1]; $0 < x < p$, $0 < y < q$:

$$u_{xx} + u_{yy} = u_{tt}, \quad u|_{t=0} = \xi(x,y), \frac{\partial u}{\partial t}|_{t=0} = \eta(x,y), \quad u|_s = 0, \tag{1}$$

where u is deviation of the membrane from its equilibrium position, which coincides with the plane x, y, and S is boundary of the rectangle $0 < x < p$, $0 < y < q$. Let the initial conditions $\{\xi(x,y), x \in [0,p], y \in [0,q]\}$, $\{\eta(x,y), x \in [0,p], y \in [0,q]\}$ be an strongly Orlicz stochastic processes.

Solving problems similar (1) we look for a solution of the form

$$u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} V_{nm}(x,y) \left[a_{nm} \cos \sqrt{\lambda_{nm}} t + b_{nm} \sin \sqrt{\lambda_{nm}} t \right],$$

$$a_{nm} = \int_0^p \int_0^q \xi(x,y) V_{nm}(x,y) dxdy, \; b_{nm} = \frac{1}{\sqrt{\lambda_{nm}}} \int_0^p \int_0^q \eta(x,y) V_{nm}(x,y) dxdy,$$

λ_{nm} and $V_{nm}(x,y)$ are eigenvalues and eigenfunctions of the Sturm-Liouville problem [1]: $V_{xx} + V_{yy} + \lambda V = 0$, $V|_s = 0$.

The conditions of existence with probability one of twice continuously differentiated solution of the boundary-value problem of Rectangular membrane's vibrations with strongly Orlicz stochastic processes are found. The estimation for distribution of supremum of this problem has been got too.

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National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine
E-mail address: aslyvka@tn.uz.ua

STABILITY IN MEAN SQUARE OF SOLUTION OF CAUCHY PROBLEM FOR LINEAR PARTIAL DIFFUSION STOCHASTIC DIFFERENTIAL EQUATIONS WITH MARKOV PERTURBATIONS

V.C. Yasinkij

On the probability base $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ the Cauchy problem for partial stochastic differential equations are determined [1]

$$\frac{\partial}{\partial t}\left[Q\left(A\left(\xi(t)\right), \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u\left(t, x, \omega\right)\right] + Q\left(B\left(\xi\left(t\right)\right), \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u\left(t, x, \omega\right) =$$

$$= \frac{\partial}{\partial t} Q\left(C\left(\xi\left(t\right)\right), \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u\left(t, x, \omega\right) \frac{dw(t)}{dt}$$

with impulse switchings $\Delta u\left(t, x, \omega\right)|_{t=t_k} = g\left(t_{k^-}, \xi\left(t_{k^-}\right), \eta_k\right), \quad \forall t_k \in \{t_n \Uparrow, n \in \mathbb{N}\}$ and with initial conditions $Q\left(A\left(\xi\left(t\right)\right), \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u\left(t, x, \omega\right)\Big|_{t=0} = [Qu]_0$. $w(t, \omega)$ is Wiener scalar process not depending from Markov process $\xi(t) \equiv \xi(t, \omega)$ with right-continuous realizations on the compact phase space Y and from Markov chain $\{\eta_{k \geq 1}\}$;the operator is defined: $Q\left(A\left(\cdot\right), q, p\right) := \sum\limits_{k=1}^{n} \sum\limits_{j=1}^{m} a_{kj}\left(\xi\left(t\right)\right) q^k p^j$, for matrixes B,C acts respectively. Under strong solution of Cauchy problem we understand random function conformed with filtration $\{\mathcal{F}_t, t \geqslant -\tau, \tau > 0\}$ and such, that with pro-
bability 1 for every (t, x) satisfy respective diffusion stochastic integral. Determine M_T - space of random functions with norm

$$\left\|u\left(t, x, \omega\right)\right\|^2 := \int\limits_0^T \mathbb{E}_u\left(t\right) dt = \ = \int\limits_0^T \mathbb{E}\left[\int\limits_{-\infty}^{+\infty} \left|u\left(t, x, \omega\right)\right|^2 dx\right]^2 dt.$$

The sufficient conditions of asymptotic stable in l.i.m. and global exponential stable in l.i.m. were obtained.

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Chernivtsi national university named after Yurij Fedkovich, Chernivtsi, Ukraine
E-mail address: yasinsk@list.ru

HÖLDER REGULARITY OF THE SOLUTIONS OF SDE WITH MIXED DRIVING

M. Zähle

We consider SDE in $\mathbb{R}^n$ with time dependent (not necessarily adapted) random coefficients, where one continuous driving process admits a generalized quadratic variation process. The other driving processes are assumed to possess sample paths in fractional Sobolev spaces of order $> 1/2$. In particular, one Brownian motion and a finite number of multifractional Brownian motions can be treated. The corresponding stochastic integrals are determined as generalized forward integrals. A higher-dimensional version of the pathwise Doss-Sussman approach to global solutions has been developed which combines the stochastic Itô-type calculus with fractional calculus via two auxiliary differential equations, one ordinary and the other fractional. This extends our former local approach. In the talk we will present optimal Hölder regularity properties of the unique solutions. (Joint work with E. Schneider.)

Mathematical Institute, University of Jena, D-07737, Jena
E-mail address: martina.zaehle@uni-jena.de

ON STOCHASTIC REPRESENTATION OF THE MOTION OF SOME VISCOUS FLUIDS

M.E. Zalygaeva

Using the theory of equations with Nelson's mean derivatives in the framework of Lagrangian approah to hydrodynamics we suggest a special stochastic description of the motion of viscous fluid on flat n-dimensional torus T^n, having its viscous term in the form of second order differential operator $\tilde{\mathbf{B}} = \frac{1}{2}\tilde{B}^{ij}\frac{\partial^2}{\partial x^i \partial x^j}$.

We suppose that there exists a constant linear operator B such that $(\tilde{B}^{ij}) = BB^*$. By $D^s(T^n)$ we denote the group of H^s-Sobolev diffeomorphisms of T^n with $s > \frac{n}{2} + 1$. Let $g(t)$ be a geodesic of Levi-Civita connection of weak Riemannian metric on $D^s(T^n)$. Recall that $g(t)$ is a flow of the so-called diffuse matter on T^n (see [1]). Introduce the vector $v(t) = \dot{g}(t) \circ g^{-1}(t)$ and construct the vector field $V(t, m) = E(v(t, m - Bw(t)))$

Theorem 1. *The vector field $V(t,m)$ satisfies the following analogue of Burgers equation:*

$$\frac{\partial V(t,m)}{\partial t} + (V(t,m)\cdot\nabla)V(t,m) - \tilde{B}V(t,m) = 0.$$

In [2] analogous result was obtained for the classical Burgers equation.

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Universitetskaya pl.1, Voronezh, 394006, Russia

E-mail address: zalygaeva@math.vsu.ru

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PROPERTIES OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH RANDOM COEFFICIENTS, NON-LIPSCHITZ DIFFUSION AND POISSON MEASURES

V.P. Zubchenko

We study properties of solutions of the following stochastic differential equation:

$$X(t) = \zeta(t) + \int_0^t a(X(s))\,ds + \int_0^t g(X(s))\,dW(s)$$
$$+ \int_0^t \int_{\mathrm{R}} q_1(X(s),y)\tilde{\nu}(ds,dy) + \int_0^t \int_{\mathrm{R}} q_2(X(s),y)\mu(ds,dy),$$

where $\zeta : \Omega\times\mathbb{R}^+ \to \mathbb{R}$ is a random process; $W(t)$ is the Wiener process; $\nu(dt,dy)$ is the Poisson measure, $E\nu(dt,dy) = \Pi(dy)dt$, $\tilde{\nu}(dt,dy) = \nu(dt,dy) - \Pi(dy)dt$ is the centered Poisson measure, $\Pi(\cdot)$ is a sigma-finite measure on the σ-algebra of the Borel sets in $\mathbb{R}$; $\mu(dt,dy)$ is non-centered Poisson measure, $E\mu(dt,dy) = m(dy)dt$, is a finite measure on the σ-algebra of Borel sets in $\mathbb{R}$.

Investigation of stochastic differential equations with additional randomness, given by a random process ζ, is of interest in connection with the study of generalized Cox-Ingersoll-Ross inerest rate model. Obtained results have practical applications in actuarial and financial mathematics, risk theory and other fields of science, where models based on stochastic differential equations are used. Most such models use equation with non-Lipschitz diffusion coefficient. Also the feature of most financial markets is "jumps" of financial indices in some moments of time.

Existence and uniqueness of strong solution of stochastic differential equation with random coefficients, non-Lipschitz diffusion and Poisson measures are proved. The probability of the solution of equation of such type to become non-positive is estimated, conditions of existence of non-negative solution for the equation of such type are pointed out. We investigate limit behavior of the integral functional of the solution of equation of such type. We study the rate of convergence and some other properties of the Euler scheme the stochastic differential equations with non-Lipschitz diffusion and centered Poisson measure.

National Taras Shevchenko University of Kyiv, The Faculty of Mechanics and Mathematics, Volodymirska 64, 01601 Kyiv, Ukraine

E-mail address: v_zubchenko@ukr.net

STOCHASTIC MODELS OF EVOLUTION SYSTEMS

ASYMPTOTIC BEHAVIOR OF SUBCRITICAL BRANCHING RANDOM WALK

E.Vl. Bulinskaya

We consider a model of catalytic branching random walk (CBRW) on integer lattice $\mathbb{Z}^d$, $d \in \mathbb{N}$, in which branching may occur at the origin only. Previous study of the model showed that CBRW may be classified as supercritical, critical and subcritical. For supercritical and subcritical CBRW, the asymptotic behavior of the total number of particles as well as of the local particles numbers was treated, as time tends to infinity (see, e.g., [1], [2] and references therein). The total size of particles population was also investigated for subcritical symmetric branching random walk on $\mathbb{Z}^d$ in [3]. However, the limit behavior of *local particles numbers* in *subcritical* CBRW on $\mathbb{Z}^d$ remained unknown. Our work completes the picture. Firstly, we establish asymptotic behavior of the probability of particles presence at an arbitrary fixed point of the lattice, as time tends to infinity. Secondly, we prove conditional limit (in time) theorem for the number of particles at any fixed lattice point given that this number is strictly positive. It turns out that these results are similar to those for the model of *critical* CBRW on $\mathbb{Z}^d$ with $d = 2$ (see [1]).

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Lomonosov Moscow State University
E-mail address: bulinskaya@yandex.ru

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INVARIANTS OF STOCHASTIC DYNAMIC SYSTEMS

V.A. Doobko

The way of equations construction for stochastic kernels of integral invariants $\rho(t;x)$:

$$\int_{R^n} \rho(t;x) \prod_{j=1}^{n} dx_j = 1, \quad \text{for all} \quad t \in [0,T],$$

for generalized Ito equations (GIE) and the equation for the stochastic first integrals $u(t;x)$ of the GIE :

$$du(t;x) = -[a_i(t)\frac{\partial}{\partial x_i}u(t;x) + \frac{1}{2}b_{i,k}(t)b_{j,k}(t)\frac{\partial^2 u(t;x)}{\partial x_i \partial x_j} -$$
$$-b_{i,k}(t)\frac{\partial}{\partial x_i}(b_{j,k}(t)\frac{\partial}{\partial x_j}u(t;x))]dt$$
$$-b_{i,k}(t)\frac{\partial}{\partial x_i}u(t;x)dw_k(t) + \int_{R(\gamma)}[u(t;x-g(t;x^{-1}(t;x;\gamma);\gamma)) - u(x;t)]\nu(dt;d\gamma),$$

where $a(t)$, $b_k(t)$, $g(t;x;\gamma) \in R^n$ – are coefficients of GIE, $x^{-1}(t;y;\gamma)$ – is the solution of equations $x + g(t;x;\gamma) = y$, $\nu(dt;d\gamma)$ – is not centered Poisson measure, $w_k(t)$, $(k = \overline{1,m})$ – are the independent processes of Wiener, and in over repeated indices summation, is described.

Establishes requirements for the class of GIE that ensure constancy in time for functionals, that associated with the evolution models in random media [1, 2].

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National Aviation University, Kiev, Ukraine
E-mail address: doobko2008@yandex.ru

ON CONTINUOUSNESS OF LINEAR FUNCTIONAL AND STOCHASTIC INTEGRALS

K.G. Dziubenko

Let $(\Omega, \mathcal{F}, P)$ be a probability space, L − linear space, τ − topology in L. Random functional φ in L is *stochastically linear* if $\varphi(\alpha_1(\omega)x_1 + \alpha_2(\omega)x_2, \omega) = \alpha_1(\omega)\varphi(x_1, \omega) + \alpha_2(\omega)\varphi(x_2, \omega)$ a.s. for all $x_1, x_2 \in L$ and all random variables $\alpha_n(\omega)$ with $|\alpha_n| \le 1$ a.s., $n = \overline{1,2}$. $Ker\varphi = (Ker\varphi)(\omega) = \{x \in L : \varphi(x, \omega) = 0\}$ is *kernel of random functional* φ in L. $Ker\varphi$ is *stochastically closed in topology* τ if for all $\{x_n, n \in N\} \subset N$ with $x_n \to \overrightarrow{0}$ in τ and for all random variables $\alpha_n(\omega)$ with $|\alpha_n| \le 1$ a.s., $n \in N$, with probability 1 $\varphi(\alpha_1(\omega)x_1 + \alpha_n(\omega)x_2, \omega) = 0$ for all $n \in N$ except finite number implies $\varphi(\alpha_1(\omega)x_1, \omega) = 0$. $X(t)$, $t \in [a, b]$, is a random process with orthogonal increments ($a < b$). A_2 is the set of all measurable $f : [a, b] \to R$ such that $r_X(f) = \left(\int_a^b (f(t))^2 dE(X(t) - X(a))^2\right)^{\frac{1}{2}} < +\infty$. Differential representation in theorem 3 describes stochastic evolution, generated by basic process $X(t)$.

Theorem 1. *Stochastically linear random functional φ in L is continuous a.s in τ if and only if $Ker\varphi$ is stochastically closed in τ.*

Theorem 2. *Random functional $\varphi(f) = \int_a^b f(t)dX(t)$, $f \in \mathrm{A}_2$, is continuous a.s. in norm $r_X(\cdot)$.*

Theorem 3. *Let $t_0 \in [a, b]$; $g(t, \cdot) \in \mathrm{A}_2$, $t \in [a, b]$; $g(t_0, \cdot)$, $g'_t(t_0, \cdot)$ are continuous at t_0; $g'_t(\cdot, s)$ are continuous at t_0 uniformly by $s \in [a, b]$. Then*

$$\int_a^{t_0+\Delta t} g(t_0 + \Delta t, s)dX(s) - \int_a^{t_0} g(t_0, s)dX(s) = \left(\int_a^{t_0} g'_t(t_0, s)dX(s)\right)\Delta t +$$

$$+ g(t_0, t_0)\Delta X(t_0) + o(\Delta t) + o(\left(E(\Delta X(t_0))^2\right)^{\frac{1}{2}}), \Delta t \to 0\, a.s.$$

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Institute of Cybernetics NASU, 40 Academician Glushkov Prospect, Kiev, 03187
E-mail address: kdz913@gmail.com

STOCHASTIC VERSUS DETERMINISTIC IN MODELLING PHYTOPLANKTON AGGREGATION

N. El Saadi[1], A. Bah[2]

Modeling the dynamics of phytoplankton is of great importance to many aspects of human interest, since phytoplankton provides the basis of the food chain in lakes, seas and oceans. A particularly interesting aspect in ecology and oceanography is the aggregation behavior in phytoplankton.

In this paper, we investigate the numerical treatment of a nonlinear Stochastic Partial Differential Equation describing phytoplankton aggregation. We present the numerical solutions for several scenarios simulated with parameters values corresponding to real conditions in nature. A comparison is made with two deterministic advection-diffusion-reaction models usually used by ecologists, to emphasize the efficiency of the stochastic equation in modeling the aggregation behavior in phytoplankton.

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[1] LAMOPS, ENSSEA (ex INPS), Algiers, Algeria
E-mail address: enadjia@gmail.com
[2] UMI 209, UMMISCO (IRD-Paris 6), ESP-UCAD, Dakar, Senegal
E-mail address: alassane.bah@gmail.com

CHOOSING AN ADEQUATE PROBABILISTIC MODEL FOR EVOLUTION SYSTEMS WITH CONTROL

M.A. Fedotkin[1], A.V. Zorine[2]

Three classical approaches [1] to construct a probabilistic model $(\Omega, \mathfrak{F}, \mathbf{P})$ for a real evolution system have a disadvantage that construction succeeds only for simple systems [1, 2]. A cybernetic approach proposed in [1] is based on selecting general properties of a system of general nature, i.e. on selecting the scheme, the information, the coordinates and the function of the system. Besides, it is necessary to define a set U with elements u. The nature of U depends on the system's structure and on the external environment in which the system operates. The cybernetic approach results in a family $M = \{(\Omega, \mathfrak{F}, \mathbf{P}_u(\cdot)) \colon u \in U\}$ of probabilistic models for the system. A problem of choosing an adequate model from M is hard. Solution to the problem is based on the choice of the optimal value v from a given set V of propositions concerning adequacy of the system's model. It is assumed that for a fixed $u \in U$ and an accepted proposition $v \in V$ we get a choice for a model with error or a loss $l(u, v)$ with values in some set Z. In this paper a method is suggested which allows to formulate and solve an optimization problem for the error function $l(u, v)$ and therefore choose an adequate model for the system. Different decision problems are considered with or without observations of the system and when u, v are either nonrandom or random [2].

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603950, Russia, Nizhni Novgorod, prospekt Gagarina, 23, N.I. Lobachevsky State University of Nizhni Novgorod, Department of computational mathematics and cybernetics

E-mail address: [1] fma5@rambler.ru, [2] zoav1602@gmail.com

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MARKOV EVOLUTION OF A SPATIAL ECOLOGICAL MODEL WITH DISPERSION AND COMPETITION

D.L. Finkelshtein

The evolution of an individual-based spatial ecological model with dispersion and competition is studied. In the model, an infinite number of individuals – point particles in $\mathbb{R}^d$ – reproduce themselves, compete, and die at random. These events are described by a Markov generator (1), which determines the evolution of states understood as probability measures on the space of particle configurations, namely

$$(LF)(\gamma) = \sum_{x\in\gamma}\Big[m + \sum_{y\in\gamma\setminus x} a^-(x-y)\Big]\,[F(\gamma\setminus x) - F(\gamma)] \tag{1}$$

$$+ \sum_{y\in\gamma}\int_{\mathbb{R}^d} a^+(x-y)\,[F(\gamma\cup x) - F(\gamma)]\,dx.$$

Here $m \geq 0$, $0 \leq a^\pm \in L^1(\mathbb{R}^d)$, $\gamma \in \Gamma$, Γ is the space of all locally finite subsets of $\mathbb{R}^d$.

The main result is a statement that the corresponding correlation functions evolve in a scale of Banach spaces and remain sub-Poissonian, and hence no clustering occurs, if the dispersion is subordinate to the competition. Namely,

Theorem 1. *Let $a^+(x) \leq \theta a^-(x)$, $x \in \mathbb{R}^d$ for some $\theta > 0$, and let $e^{\alpha^*}\theta < 1$, $m > \int_{\mathbb{R}^d} a^+(x)dx$. Then there exists an evolution of probability measures on Γ, s.t.*

$$\frac{d}{dt}\int_\Gamma F(\gamma)d\mu_t(\gamma) = \int_\Gamma LF(\gamma)d\mu_t(\gamma) \tag{2}$$

on the finite time interval $[0, T^*)$, such that any $t \in [0, T^*)$ and for any $n \in \mathbb{N}$ the n-th order correlation function $k_t^{(n)}$ of the measure μ_t satisfies the following inequality $k_t^{(n)}(x_1, \ldots, x_n) \leq A_t C_t^n$, $x_1, \ldots, x_n \in \mathbb{R}^d$ for some $A_t, C_t > 0$.

These results are essential extensions of results obtained in [1].

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Institute of Mathematics, NAS of Ukraine, 01601, Kiev-4, Ukraine

E-mail address: fdl@imath.kiev.ua

SPATIAL AUTOCORRELATION FOR SUBDIVIDED POPULATIONS WITH INVARIANT MIGRATION SCHEMES

Ola Hössjer

We consider a population whose genetic composition varies between subpopulations and in time. It is then helpful to extract a few statistics the summarize the spatio-temporal distribution of alleles at one or several polymorphic loci. In [1] and [2] a new quasi equilibrium approach was introduced for computing the variance effective population size N_{eV} and the fixation index F_{ST}. The main idea is to view the vector of allele frequencies at all subpopulations as a vector valued autoregressive process.

In order to compute N_{eV} and F_{ST} one must know the matrix of migration rates between subpopulations as well as the genetic drift covariance matrix, which depends on how the population reproduces from one generation to the next. We show that when migration is translationally invariant, the discrete Fourier transform can be used to define a very fast algorithm for computing not only N_{eV} and F_{ST}, but also a correlation function, which quantifies the pairwise genetic similarity between subpopulations.

Our findings are illustrated for the island model, the hierarchical island models, various steppings stone models and isolation by distance models.

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Department of Mathematics, Stockholm University, SE 106 91 Stockholm
E-mail address: ola@math.su.se

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NETWORK MODELS WITH INTERDEPENDENT INPUT IN HEAVY TRAFFIC

E. Lebedev, A. Chechelnitsky

The process of information treatment which is an object of our interest is a vector of large dimension with complex systems of stochastic relations which determine the process. In previous works, as a rule, input flows were independent. Thus, interdependence of service process components was caused by trajectory intersections for information packets before their outputs from the network. In the paper we omit this restriction and the process of information treatment became more complicated.

That is why the method of functional limit theorems is especially efficient for the analysis of information treatment in stochastic networks under consideration. The method gives the possibility to find those principles which form the foundation of a functioning process for a network of the given type, to construct an approximate process and to calculate the distribution of functionals in order to obtain integral characteristics for the functioning process.

By virtue of conditions imposed on model parameters in our case an approximate process will be diffusion and we will deal with development of the method of diffusion approximation for multi-channel stochastic networks. We consider a multi-channel queuing system with multi-dimensional Poisson input flow.

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Kyiv Taras Shevchenko National University, Cybernetics, av.Glushkowa 4-D, Kiev, Ukraine
E-mail address: alex_che@voliacable.com

PIOSSON APPROXIMATION OF PROCESSES WITH INDEPENDENT INCREMENTS WITH MARKOV SWITCHING

N. Limnios[1], I.V. Samoilenko[2]

The additive functional $\xi^\varepsilon(t), t \geq 0, \varepsilon > 0$ on $\mathbb{R}^d$ in the series scheme with small series parameter $\varepsilon \to 0$, $(\varepsilon > 0)$ is defined by the stochastic additive functional

$$\xi^\varepsilon(t) = \xi_0^\varepsilon + \int_0^t \eta^\varepsilon(ds; x(s/\varepsilon)).$$

The family of the Markov jump processes with *independent increments* $\eta^\varepsilon(t;x), t \geq 0,\ x \in E$ on $\mathbb{R}^d$, is defined by the generators

$$\mathbf{\Gamma}^\varepsilon(x)\varphi(u) = b_\varepsilon(u;x)\varphi'(u) + \frac{1}{2}c_\varepsilon(u;x)\varphi''(u) + \int_{\mathbb{R}^d} [\varphi(u+v) - \varphi(u) - v\varphi'(u)\mathbf{1}_{(|v|\leq 1)}]\Gamma^\varepsilon(u, dv; x),$$

where $b_\varepsilon(u;x)=\int_{\mathbb{R}^d} v\Gamma^\varepsilon(u,dv;x)$, $c_\varepsilon(u;x)=\int_{\mathbb{R}^d} vv^*\Gamma^\varepsilon(u,dv;x)$, and $\Gamma^\varepsilon(u,dv;x)$ is the intensity kernel, satisfying the conditions $\Gamma^\varepsilon(u,\{0\};x)=0, \int_{|v|\leq 1} vv^*\Gamma^\varepsilon(u,dv;x)<\infty$.

The switching Markov process $x(t), t\geqslant 0$ on the standard phase space $(E,\mathcal{E})$, is ergodic and defined by the generator $\mathbf{Q}\varphi(x)=q(x)\int_E P(x,dy)[\varphi(y)-\varphi(x)]$.

Theorem 1. *Under Poisson approximation conditions the weak convergence takes place* $\xi^\varepsilon(t)\Rightarrow\xi^0(t), \quad \varepsilon\to 0.$
The limit process $\xi^0(t), t\geq 0$ *is defined by the generator*

$$\overline{\mathbf{\Gamma}}\varphi(u)=\widehat{b}(u)\varphi'(u)+\int_{\mathbb{R}^d}[\varphi(u+v)-\varphi(u)-v\varphi'(u)\mathbf{1}_{(|v|\leq 1)}]\widehat{\Gamma}(u,dv),$$

where the average deterministic drift and the average intensity kernel are defined by the stationary distribution of the switching Markov process.

[1] Laboratoire de Mathématiques Appliquées, Université de Technologie de Compiègne, France
E-mail address: nikolaos.limnios@utc.fr
[2] 01601, IM NASU, Tereschekivska str. 3, Kyiv, Ukraine
E-mail address: isamoil@imath.kiev.ua

STATISTICAL MECHANICS OVER THE CONE OF DISCRETE MEASURES

Tatiana Pasurek

We construct Gibbs perturbations of the Gamma process in $\mathbb{R}^d$, which may be used in applications to model systems of densely distributed particles (like, e.g., ecological systems in the presence of biological diversity). We propose a definition of Gibbs states over the cone $\mathbb{K}(\mathbb{R}^d)$ of discrete Radon measures on $\mathbb{R}^d$ and analyze conditions for their existence and uniqueness. Our approach works also for general Lévy processes instead of Gamma measures. To this end, we need only the assumption that the first two moments of the involved Lévy intensity measure are finite. Also uniform moment estimates are obtained, which are essential for the construction of related diffusions. Moreover, we prove a Mecke type characterization for the Gamma measures on the cone and an FKG inequality for them. Based on the joint work with D. Hagedorn, Yu. Kondratiev and M. Röckner [2].

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Faculty of Mathematics, Bielefeld University, D 33615 Bielefeld, Germany
E-mail address: tpasurek@math.uni-bielefeld.de

NON-MARKOVIAN RANDOM MOTION IN HIGHER DIMENSIONS

Anatoliy Pogorui

The talk deals about random motion with a nonconstant velocity and uniformly distributed directions where the direction alternations occur according to renewal epochs of general distribution. We derive the renewal equation for the characteristic function of the transition density of the multidimensional motion. By using this renewal equation, we study the probability density function and the behavior of this function near the sphere of its singularity.

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Zhytomyr State Franko University, Shelushkova, 10, 222, Zhytomyr, Ukraine
E-mail address: pogor@zu.edu.ua

LARGE DEVIATIONS FOR RANDOM EVOLUTIONS WITH INDEPENDENT INCREMENTS IN THE SCHEME OF LÉVY APPROXIMATION

I.V.Samoilenko

One of the methods in large deviation theory is based on the studying of exponential martingale characterization: $\widetilde{\mu}_t=\exp\{\varphi(x(t))-\varphi(x(0))-\int_0^t \mathbb{H}\varphi(x(s))ds\}$ is the martingale. Here exponential nonlinear operator $\mathbb{H}\varphi(x):=e^{-\varphi(x)}\mathbb{L}e^{\varphi(x)}, \ \varphi(x)\in\mathcal{B}_E$.

The exponential operator in the series scheme with a small series parameter $\varepsilon\to 0$ $(\varepsilon>0)$ has the form:

$$\mathbb{H}^\varepsilon\varphi(x):=e^{-\varphi(x)/\varepsilon}\varepsilon\mathbb{L}^\varepsilon e^{\varphi(x)/\varepsilon},$$

where the operators $\mathbb{L}^\varepsilon, \varepsilon > 0$ define some Markov processes $\zeta^\varepsilon(t), t \geq 0, \varepsilon > 0$ in the series scheme on the standard phase-space $(G, \mathcal{G})$. In our case Markov processes $\zeta^\varepsilon(t), t \geq 0, \varepsilon > 0$ are the two-component processes $\xi^\delta_\varepsilon(t), \eta^\delta_\varepsilon(t), t \geq 0, \varepsilon, \delta > 0$:

$$\xi^\delta_\varepsilon(t) = \xi^\delta_\varepsilon(0) + \int_0^t \eta^\delta_\varepsilon(ds; x(s/\varepsilon^3)), \ t \geq 0, \tag{1}$$

where $\eta^\delta_\varepsilon(t) = \varepsilon\eta^\delta(t/\varepsilon^3)$ is the process with independent increments given by

$$\Gamma^\delta_\varepsilon(x)\varphi(u) = \varepsilon^{-3}\int_{\mathbb{R}}[\varphi(u+\varepsilon v) - \varphi(u)]\Gamma^\delta(dv; x), \ x \in E,$$

here $\varepsilon, \delta \to 0$ so that $\varepsilon^{-1}\delta \to 1$. The intensity kernel $\Gamma^\delta(dv; x)$ satisfies Lévy approximation conditions. Our aim is to verify the convergence of the exponential (nonlinear) generator that defines large deviations:

$$\mathbb{H}^{\varepsilon,\delta}\varphi^\delta_\varepsilon(u, x) \to H^0\varphi(u), \varepsilon, \delta \to 0, \varepsilon^{-1}\delta \to 1.$$

Theorem 1. *Solution of large deviation problem for random evolution (1) defined by a generator of two-component Markov process* $\mathbb{L}^\delta_\varepsilon\varphi(u, x) = \varepsilon^{-3}Q\varphi(\cdot, x) + \Gamma^\delta_\varepsilon(x)\varphi(u, \cdot)$, *is realized by the exponential generator*

$$H^0\varphi(u) = (\widetilde{a} - \widetilde{a}_0)\varphi'(u) + \frac{1}{2}\sigma^2(\varphi'(u))^2 + \int_{\mathbb{R}}[e^{v\varphi'(u)} - 1]\widetilde{\Gamma}^0(dv).$$

01601, IM NASU, Tereschekivska str. 3, Kyiv, Ukraine
E-mail address: isamoil@imath.kiev.ua

STOCHASTICITY AND PRINCIPLE OF DETERMINANCY

V. N. Turchyn, R.V. Lykhnenko, I.V. Ilnitsky

One of classic mechanics principles is principle of determinancy according to which the initial position of a mechanical system uniquely defines its evolution in time. But there exist mechanical systems which evolution cannot be foreseen (in the sense of classic mechanics) although their movement is described by a system of ordinary differential equations.

An example which illustrates the above said and sheds light on the question “How is it possible?” is the following experiment: a square is tossed in a plane, it collides with a horizontal surface and the side which will be at the top after the last collision with the surface is registered. Movement of the square is described by a system of ordinary differential equations. According to the unicity theorem the trajectory of the square's movement and the side which will be at the top are uniquely determined by given initial conditions (the square's angular velocity, height of tossing). But experiment shows that it's impossible to predict which of the four square's sides (1, 2, 3, 4) will be at the top.

Let's split the set of initial conditions into subsets: initial conditions which lead to outcome 1, outcome 2, outcome 3, outcome 4, and, finally, which don't lead to any of outcomes 1, 2, 3, 4 (we'll obtain a diagram of initial states).

It turns out that the diagram of initial states has fractal structure which explains impossibility of prediction of the side which will be at the top.

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Oles Honchar Dnipropetrovsk National University, Gagarin av. 72, Dnipropetrovsk, Ukraine
E-mail address: vnturchyn@gmail.com

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КИЇВСЬКИЙ НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ ІМЕНІ ТАРАСА ШЕВЧЕНКА

МІЖНАРОДНА КОНФЕРЕНЦІЯ

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